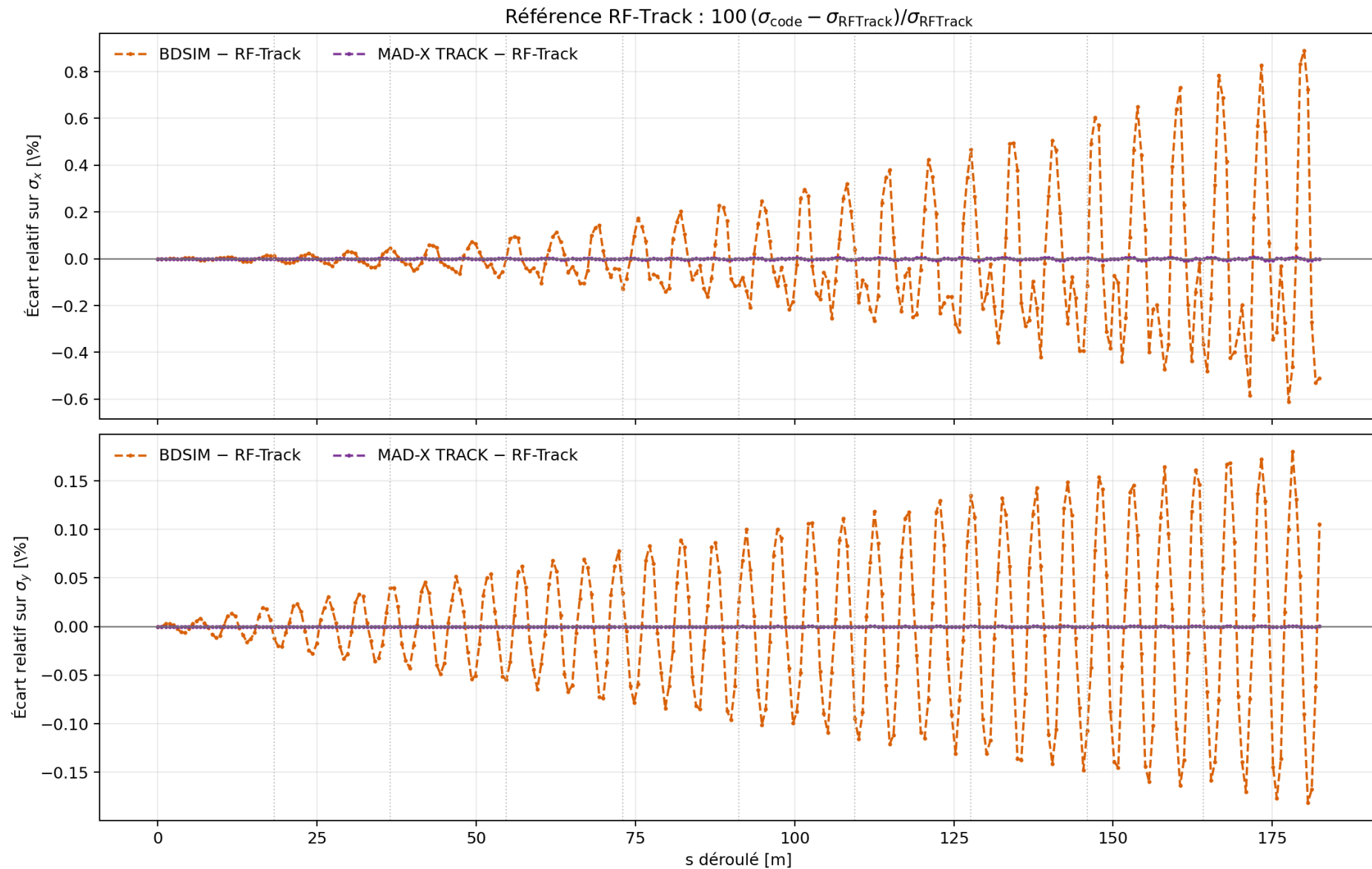


Comparison of the transverse beam envelopes obtained with MAD-X TRACK, RF-Track, and BDSIM over 15 turns of the FFA lattice:

- MAD-X TRACK vs RF-Track: **good agreement**. Both perform particle-by-particle tracking and include momentum-dependent transport.
- BDSIM vs RF-Track: relative discrepancy below 2%, **increasing progressively** with the number of turns.
- BDSIM requires the S-bend edge angles to be $< \pi/4$. The FFA edge focusing was implemented using **manually defined thin linear transfer matrices**.
- A single-dipole test shows that the **residual discrepancy in the dispersive terms R_{16} and R_{26}** is already present **in the body of the bend**, approximately (0.035%) per dipole and is not primarily caused by the thin edge matrices. This small systematic difference accumulates over successive bends and turns.

$$M_{\text{edge}}(e) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ h \tan(e) & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -h \tan(e) & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad h = \frac{\theta}{L}.$$

Relative differences



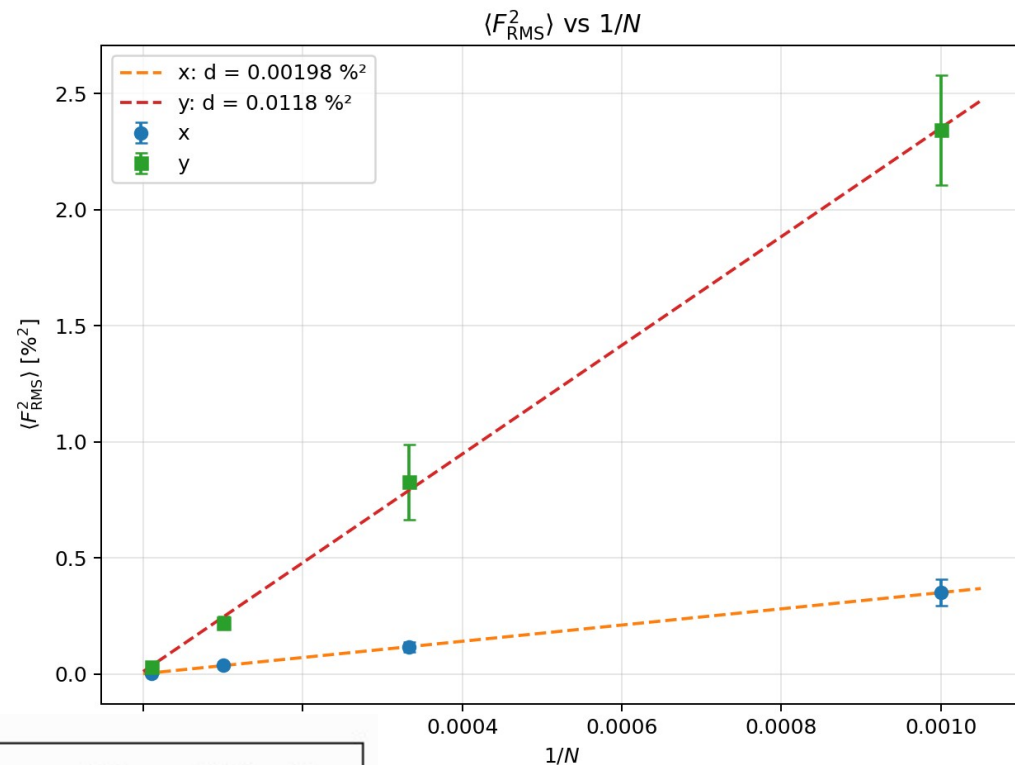
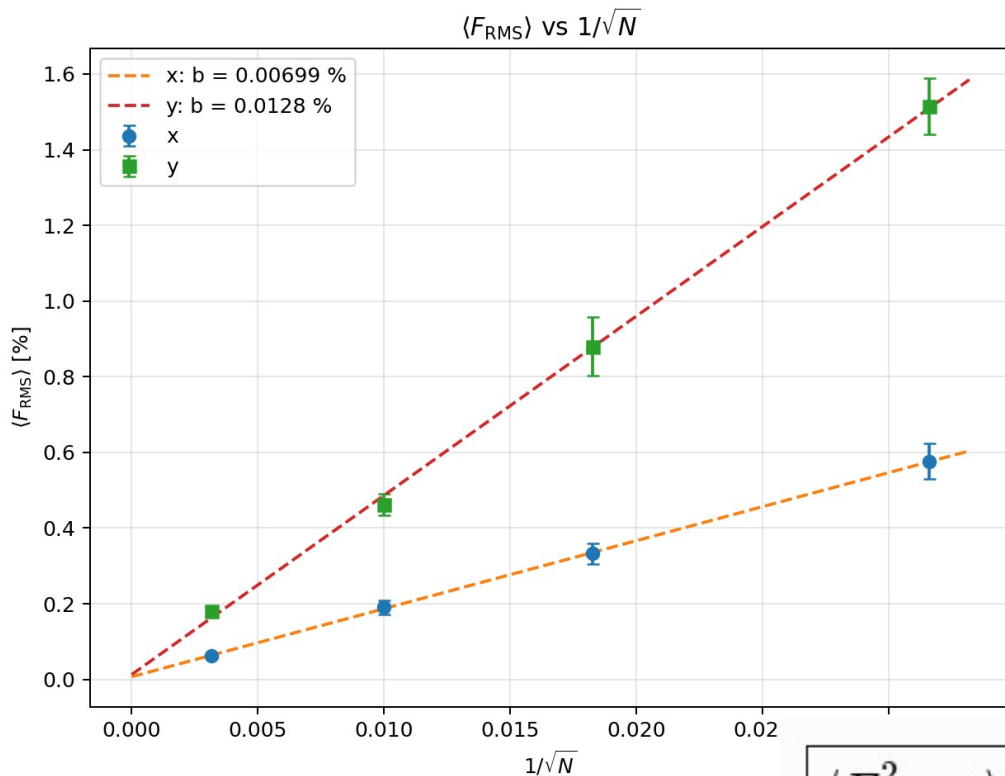
Source of remaining beam size oscillations over 100 turns in the FFA with momentum spread of 0.619 % but no space charge :

Number of macroparticles compared : [1 000, 3 000, 10 000, 100 000]

Oscillations amplitude due to limited statistics :

$$F_x = 100.0 * (\sigma_x / \text{mean}(\sigma_x) - 1.0)$$

$$f_{x,s}(n) = 100 \left(\frac{\sigma_{x,s}(n)}{\bar{\sigma}_{x,s}} - 1 \right)$$

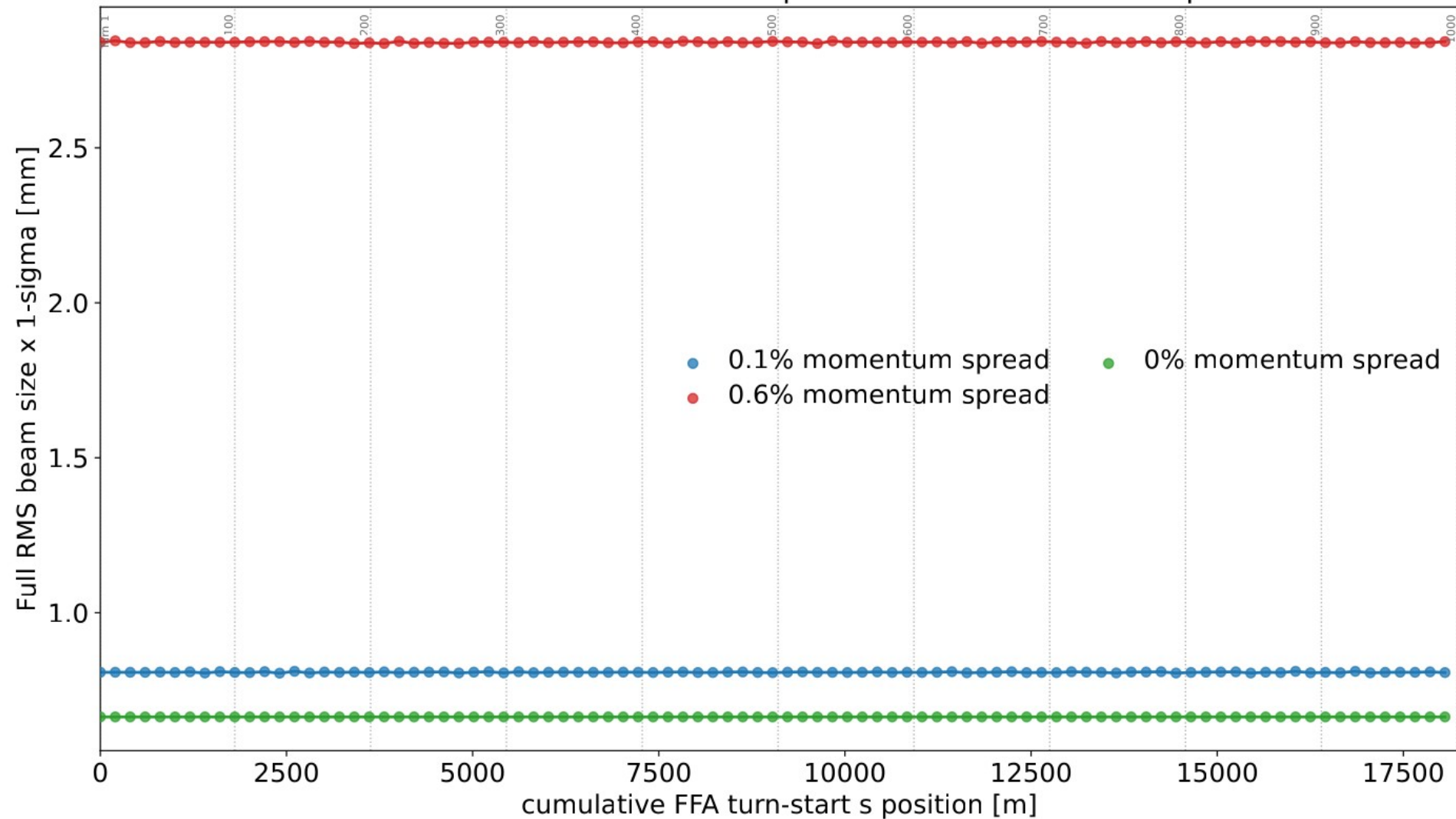


$$\langle F_{\text{RMS},x}^2 \rangle_s = \langle \text{Var}_n(f_{x,s}) \rangle_s$$

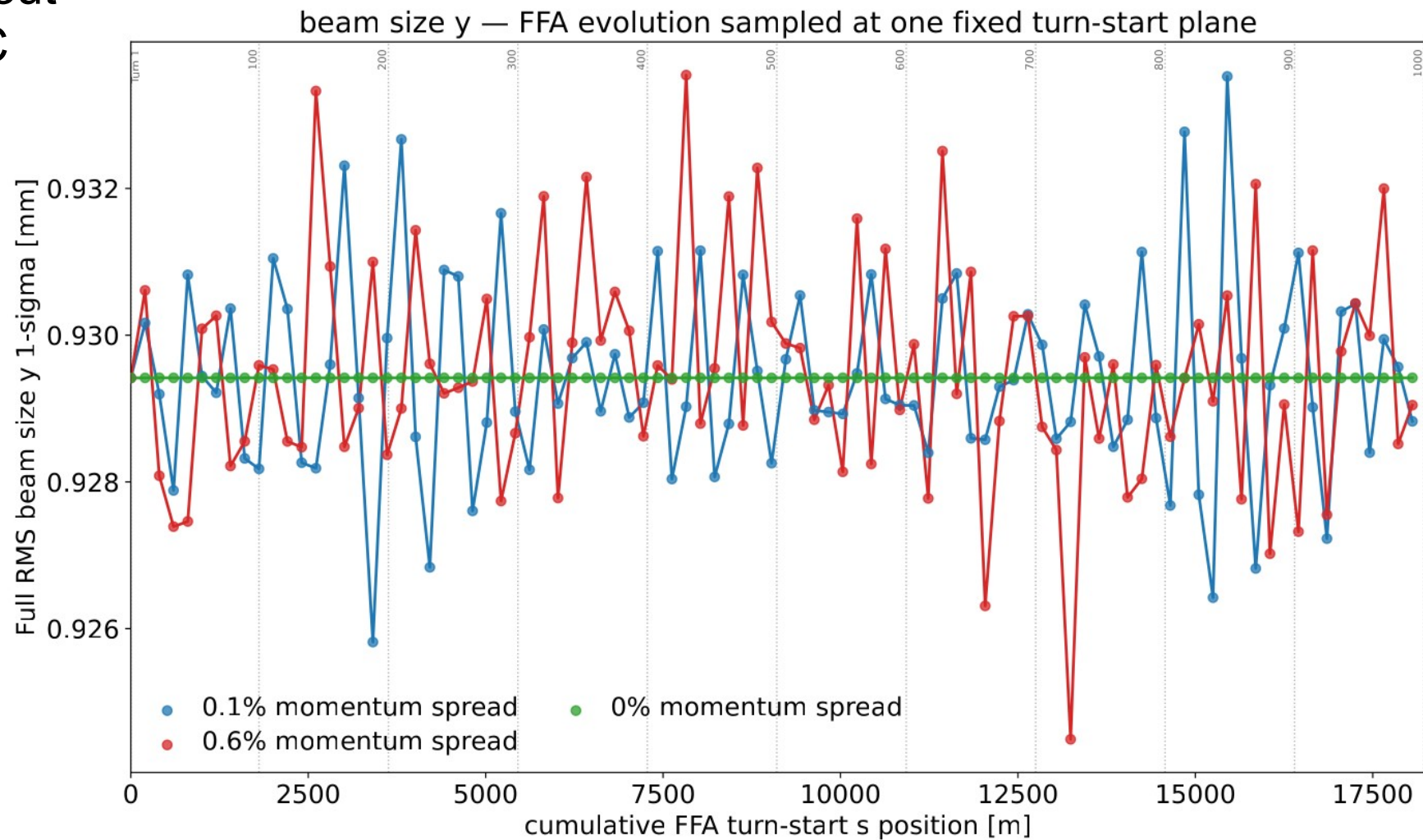
Results with new matched Twiss parameters
over 1000 turns in the FFA without space
charge ($1e5$ macroparticles)

Without SC

beam size x — FFA evolution sampled at one fixed turn-start plane



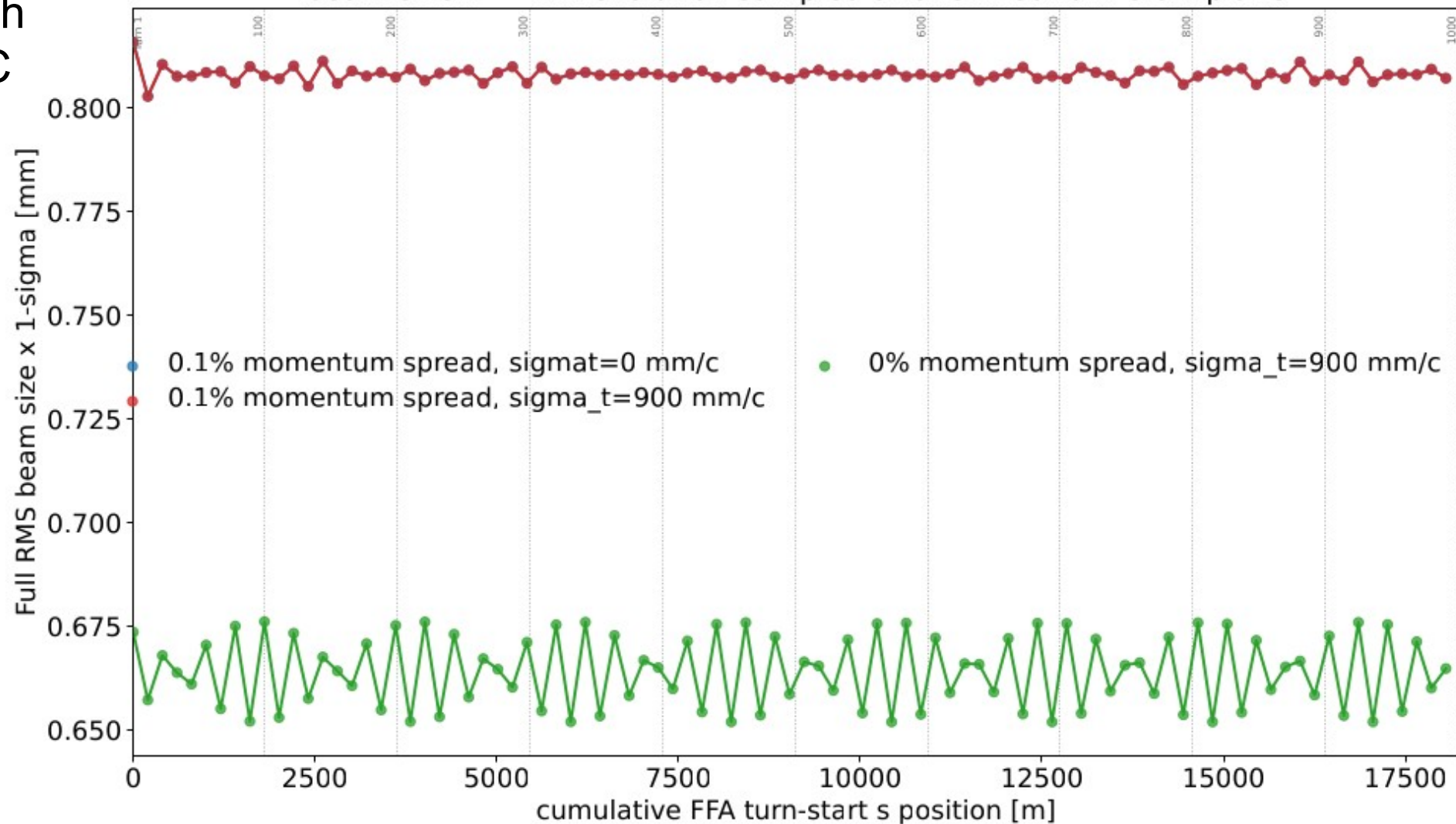
Without
SC



Results with matched Twiss parameters for $1e9$ population, $1e5$ macroparticles and space charge ON

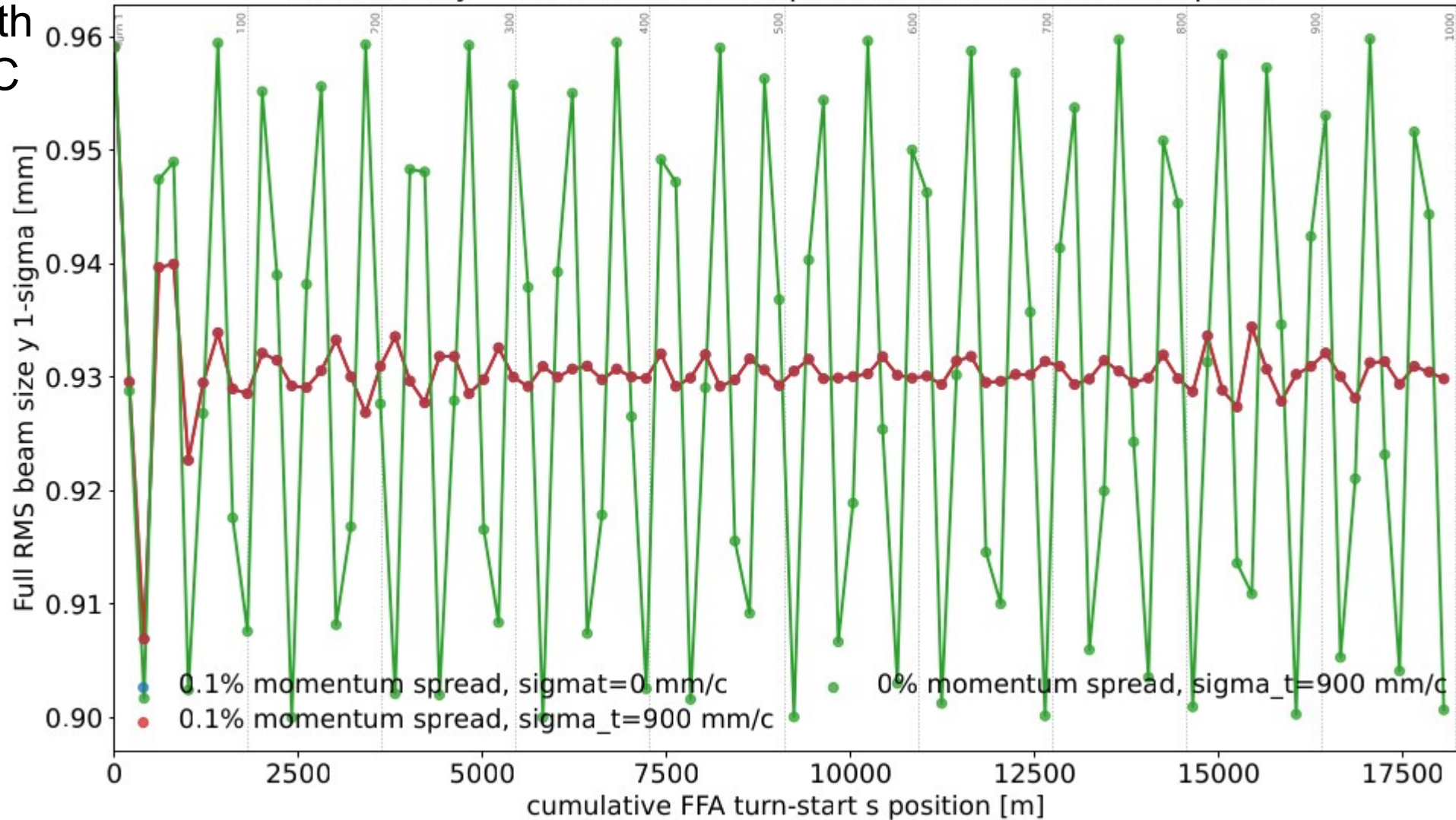
With
SC

beam size x — FFA evolution sampled at one fixed turn-start plane

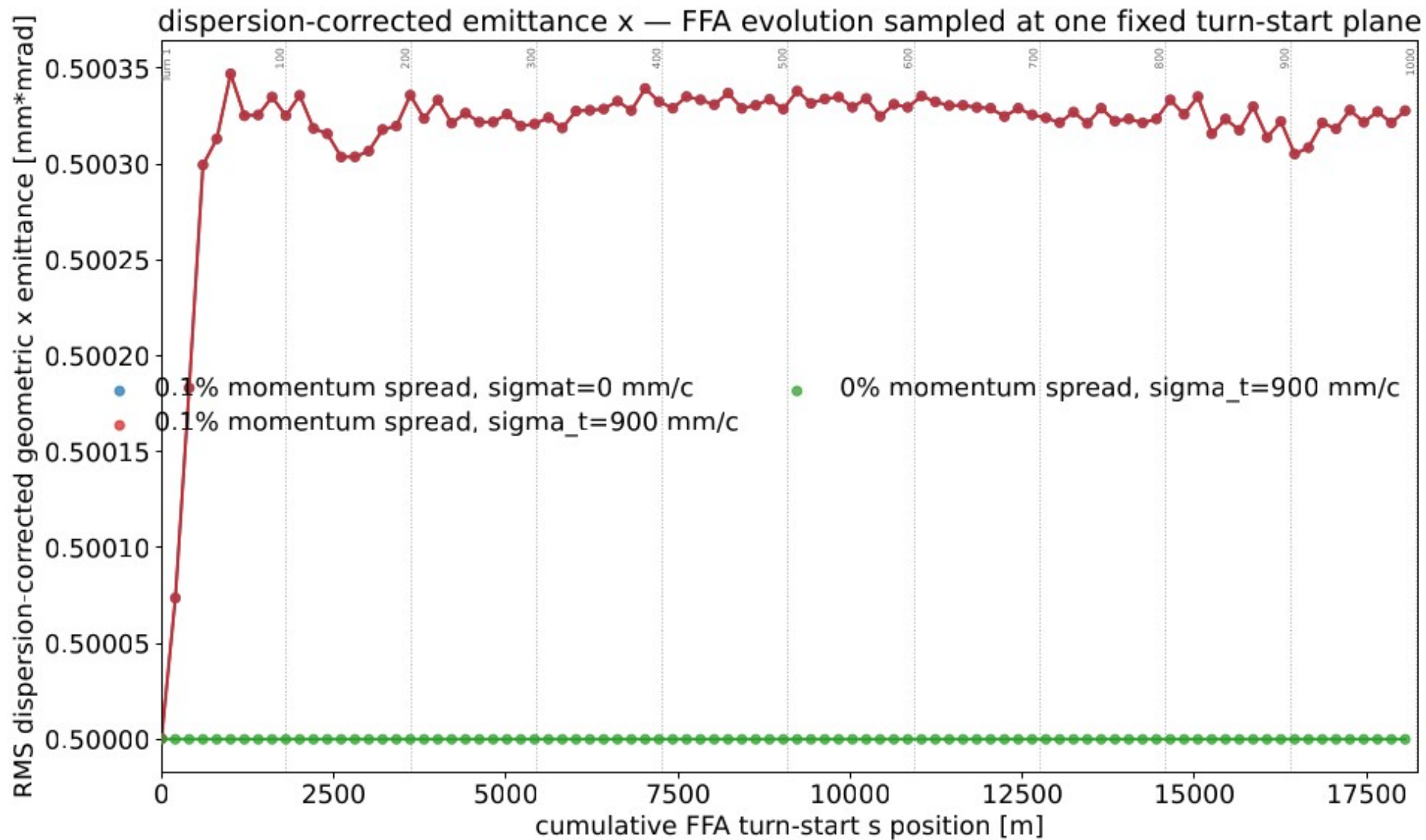


With
SC

beam size y — FFA evolution sampled at one fixed turn-start plane



With
SC



dispersion-corrected emittance y — FFA evolution sampled at one fixed turn-start plane

With
SC

