Confinement of non-neutral plasmas

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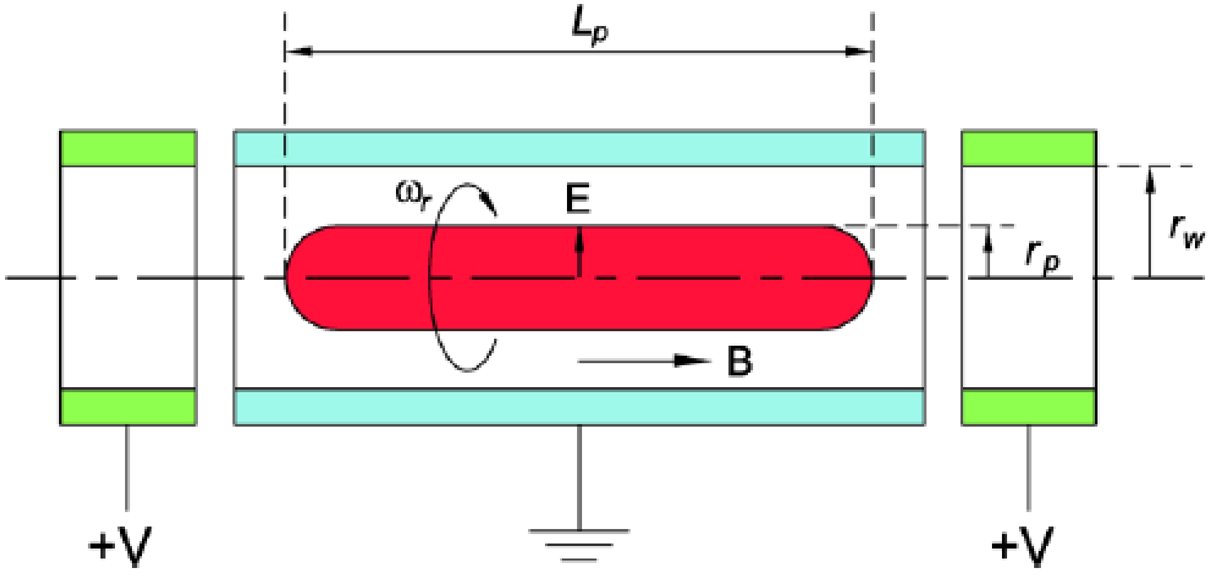
PM trap and plasma parameters



10.1103/RevModPhys.87.247

electron density	n_e
electron temperature	T
thermal velocity	v_T
plasma frequency	$\omega_{pe}^2 = n_e e^2/\varepsilon_0 m_e$
cyclotron frequency	$\omega_{ce} = eB/m_e$
gyroradius	$r_c = v_T/\omega_{ce}$
bounce frequency	$f_b = v_T/2L_p$

Plasma regime defined by $\begin{cases} \lambda_D < L_p \\ n_e(\lambda_D)^3 > 1 \end{cases}$

$\lambda_D = \sqrt{\frac{\varepsilon_0 k_B T}{n_e e^2}}$ (Debye length)

Cold-fluid equilibrium

- Momentum balance equation for a cold fluid (at equilibrium, neglecting pressure)

$$\boxed{mn\mathbf{v}\cdot\nabla\mathbf{v} = qn(\mathbf{E}+\mathbf{v}\times\mathbf{B})} \longrightarrow q(E_r + v_\theta B) + \frac{mv_\theta^2}{r} = 0 \longrightarrow -\omega_{re}^2 = \frac{1}{2}\omega_{pe}^2 - \omega_{re}\omega_{ce}$$

force balance

$$\omega_{pe}^2 = n_e e^2/\varepsilon_0 m_e$$

$$\omega_{ce} = eB/m_e$$

ω_{re} - angular velocity of plasma

$$\boxed{\omega_{re} = \omega_{re}^{\pm} \equiv \frac{1}{2}\omega_{ce}\left\{1 \pm \left(1 - \frac{2\omega_{pe}^2}{\omega_{ce}^2}\right)^{1/2}\right\}}$$

($\omega_{re}^{\pm}$ independent of r) **rigid rotation**

- $E(r)$ from Poisson equation

$$\frac{1}{r}\frac{\partial}{\partial r}[rE(r)] = -\frac{e}{\varepsilon_0}n_e(r)$$

Cold-fluid equilibrium

$$\omega_{re} = \omega_{re}^{\pm} \equiv \frac{1}{2}\omega_{ce}\left\{1 \pm \left(1 - \frac{2\omega_{pe}^2}{\omega_{ce}^2}\right)^{1/2}\right\}$$

rigid rotation

- For sufficiently large density

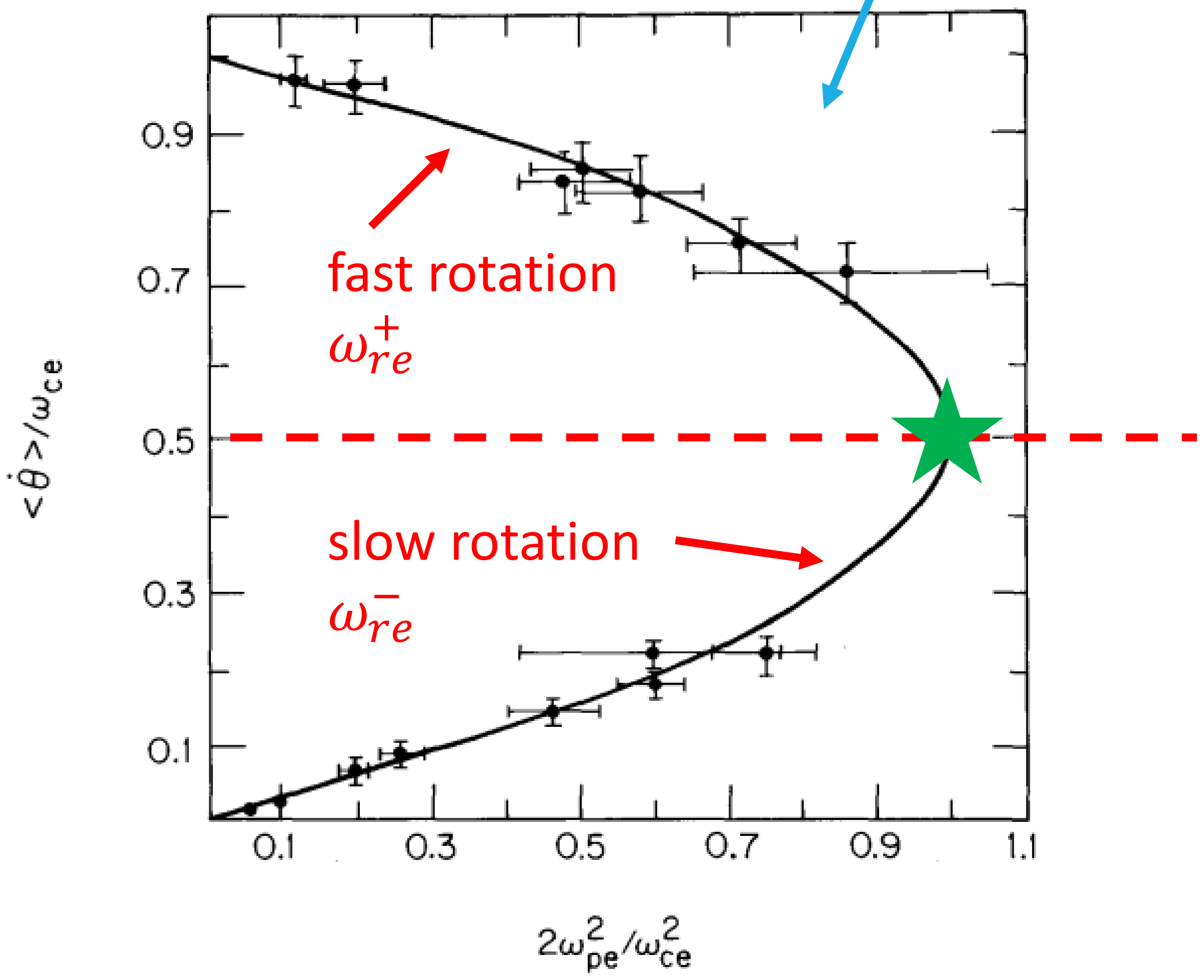
$$\omega_{re}^{\pm} = \frac{1}{2}\omega_{ce}, \text{ for } 2\omega_{pe}^2/\omega_{ce}^2 = 1$$

★ Brillouin density limit

$$n_B = 4.8 \times 10^{18}\ \mathrm{m}^{-3}\left(\frac{B}{1\ \mathrm{T}}\right)^2 \quad \text{(positrons)}$$

space-charge force too large

Physics of Nonneutral Plasmas



https://doi.org/10.1142/p251

Cold-fluid equilibrium

$$\boxed{\omega_{re} = \omega_{re}^{\pm} \equiv \frac{1}{2}\omega_{ce}\left\{1 \pm \left(1 - \frac{2\omega_{pe}^2}{\omega_{ce}^2}\right)^{1/2}\right\}}$$

- At low electron density or high magnetic field, ω_{re}^{-} can be approximated by

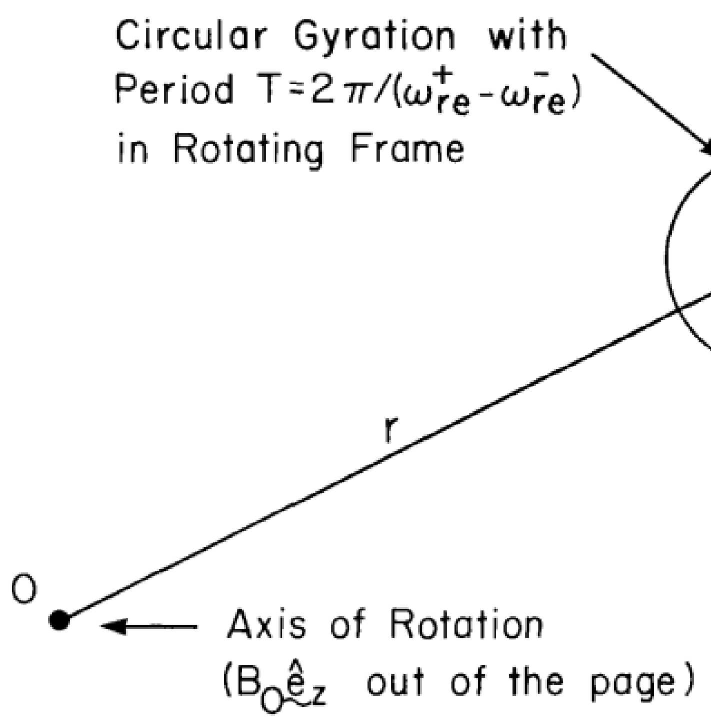
$$\omega_{re}^{-} = \frac{\omega_{pe}^2}{2\omega_{ce}} \qquad f_E = \frac{e n_e}{4\pi\varepsilon_0 B}$$

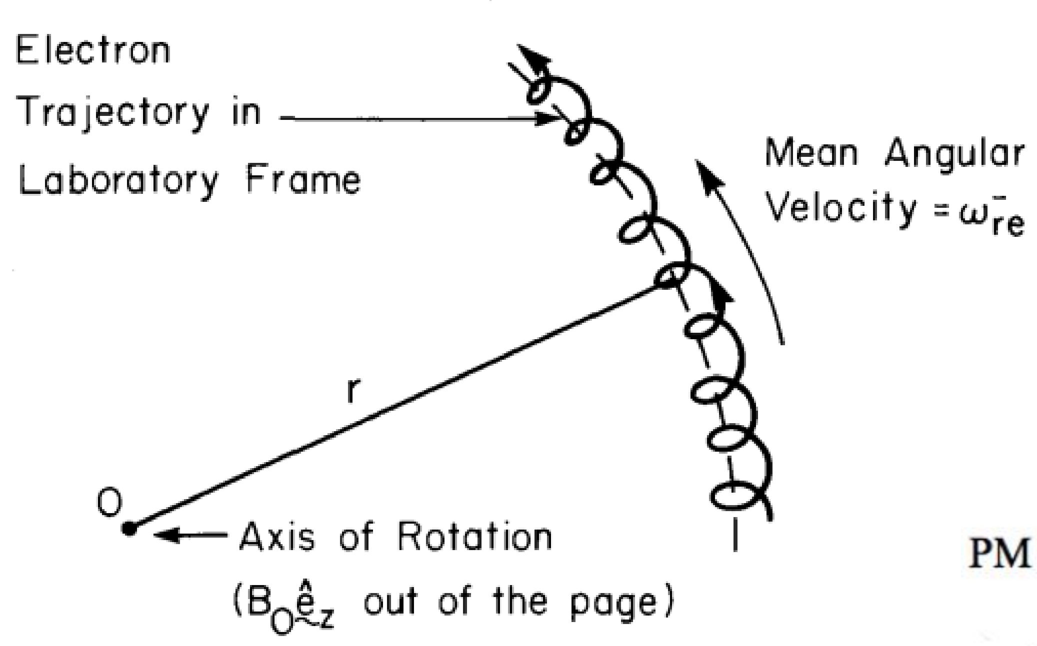
(slow $E \times B$ rotation frequency)

- Guiding centre **drift** due to a general constant force

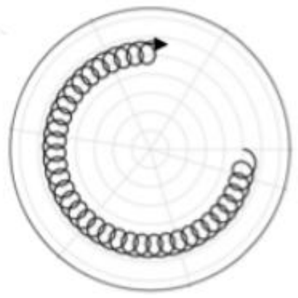
$$\mathbf{v}_\mathrm{d} = \frac{\boldsymbol{\omega}_\mathrm{c} \times \mathbf{F}}{m\omega_\mathrm{c}^2} = \frac{1}{q}\frac{\mathbf{F} \times \mathbf{B}}{B^2}$$

Single-particle trajectories





PM trap end view



Good for
$v_E \ll v_T$

- Typically, radius $\rho \ll \lambda_D, r_p$ so the particles are "tied to the field lines"

https://doi.org/10.1142/p251

Longitudinal confinement

- Plasma screens out the axial electric field by creating a space-charge potential ϕ_p

$\phi = \phi_p + \phi_{ext}$ (independent of z)

$$E_r = \frac{qnr_p^2}{2\varepsilon_0 r}, \qquad r > r_p$$

- Assuming that $\phi(r_w) = 0$ and integrating the radial electric field

$$\phi(r) = \frac{qnr_p^2}{4\varepsilon_0}\left[1 + 2\ln\left(\frac{r_w}{r_p}\right)\right] - \frac{qnr^2}{4\varepsilon_0}, \qquad r < r_p$$

min required trap voltage

- Potential difference between the wall and $r = 0$ is

$$\Delta\phi = \frac{qnr_p^2}{4\varepsilon_0}\left[1 + 2\ln\left(\frac{r_w}{r_p}\right)\right]$$

$$\boxed{\Delta\phi = 2.2 \times 10^{10}\ \mathrm{V}\left(\frac{B}{1\ \mathrm{T}}\frac{r_p}{1\ \mathrm{m}}\right)^2}$$

(at the Brillouin density limit)

Finite temperature and thermal equilibria

$$\boxed{mn\mathbf{v}\cdot\nabla\mathbf{v} = qn(\mathbf{E}+\mathbf{v}\times\mathbf{B}) - \nabla p}$$

$p = nk_BT$

Uniform temperature T

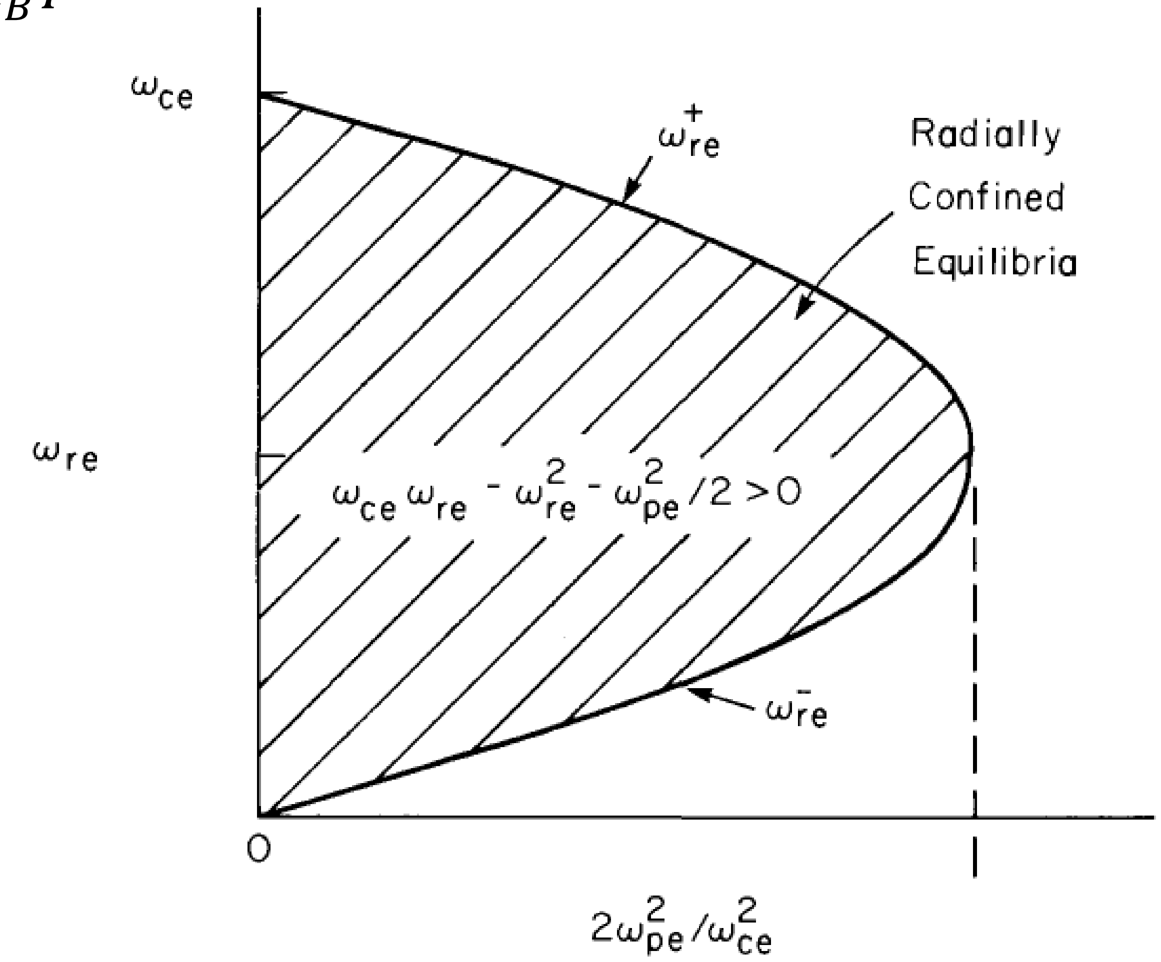
Viscosity acts to remove shears in ω_r

$$\boxed{n(r,z) = Ce^{-q\phi_{\text{eff}}(r,z)/k_BT}}$$

$$q\phi_{\text{eff}}(r,z) = \tfrac{1}{2}m\omega_r(\Omega_c - \omega_r)r^2 + q\phi(r,z)$$

$$\boxed{\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{\partial^2\phi}{\partial z^2} = -\frac{n(r,z)}{\varepsilon_0}}$$

Poisson-Boltzmann system



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Finite temperature and thermal equilibria

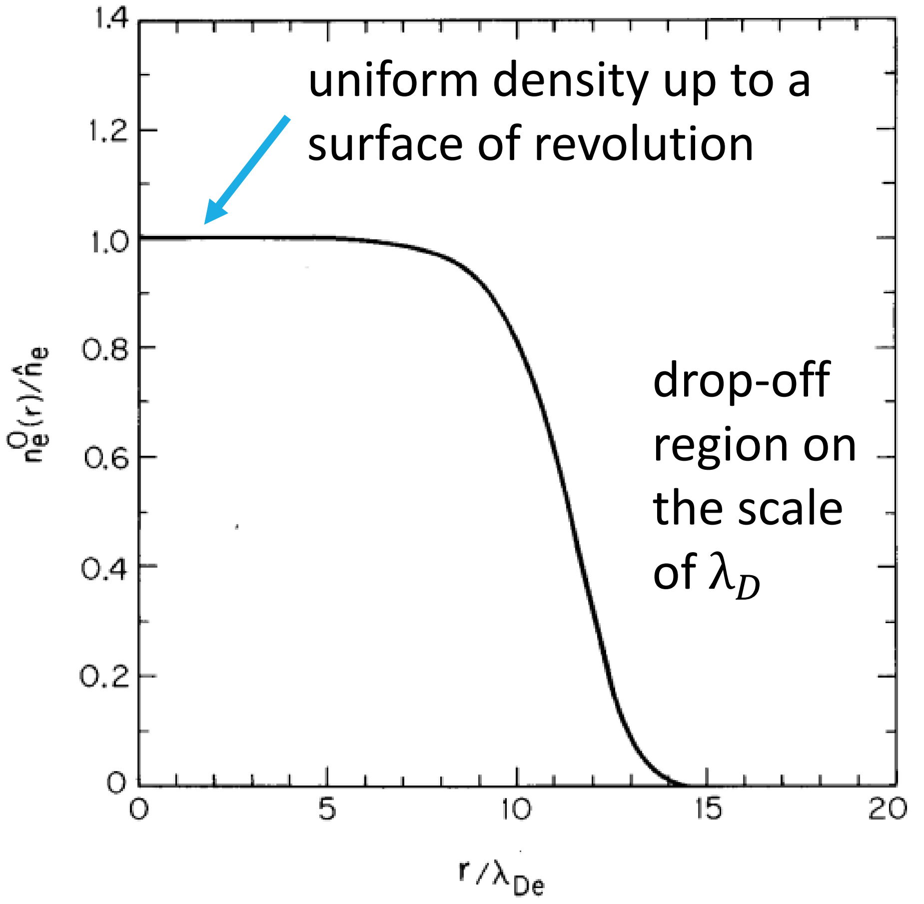
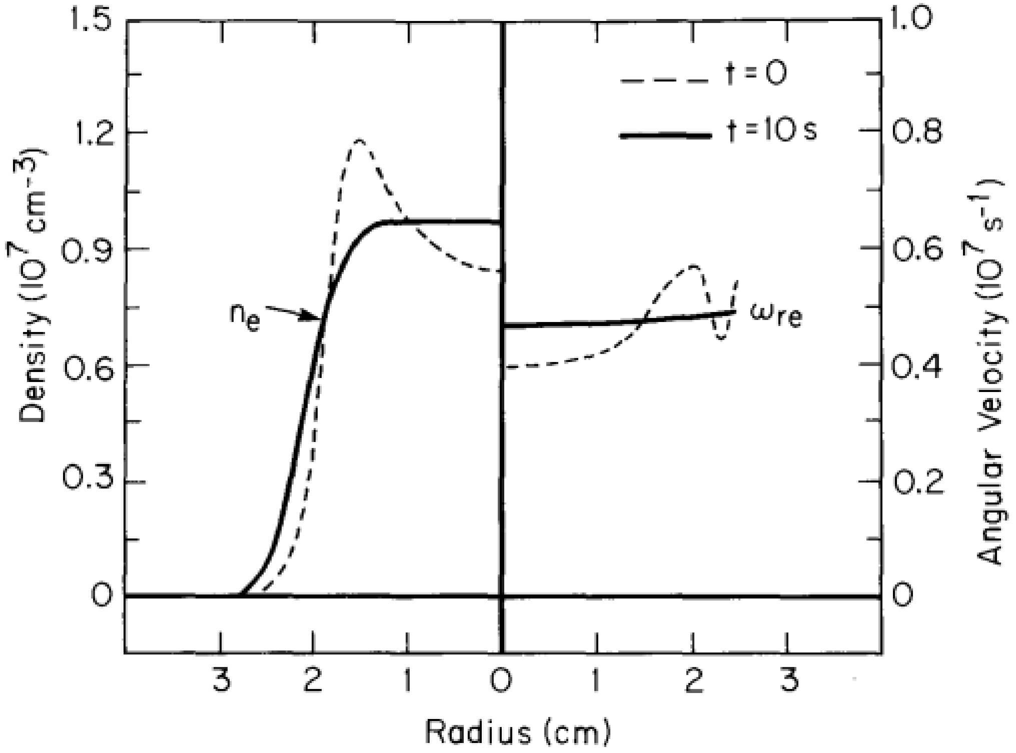


Figure 3.10. Plot of the thermal equilibrium density profile $n_e^0(r)$ versus r/λ_{De} calculated numerically from Eqs.(3.29) and (3.30) for $2\omega_{pe}^2/\omega_{ce}^2 = 0.5361$ and $\omega_{re}/\omega_{re}^- - 1 = 0.938 \times 10^{-4}$.

https://doi.org/10.1142/p251



- Non-neutral plasmas can relax to global thermal equilibrium (through binary collisions) and still remain confined.

- Relaxation occurs up to 5000 times faster than predicted by theory (doi: 10.1103/PhysRevLett.60.1290)

Constants of the motion

- Canonical angular momentum for a particle

$$p_\theta = m v_\theta r + \frac{qB}{2} r^2$$

Constant if
  - external field and potential are cylindrically symmetric
  - absence of other force with θ component (e.g., collisional drag on neutral gas)

- For a sufficiently large magnetic field

$$\boxed{P_\theta \cong \frac{qB}{2} \sum_j r_j^2}$$

Cross-field transport

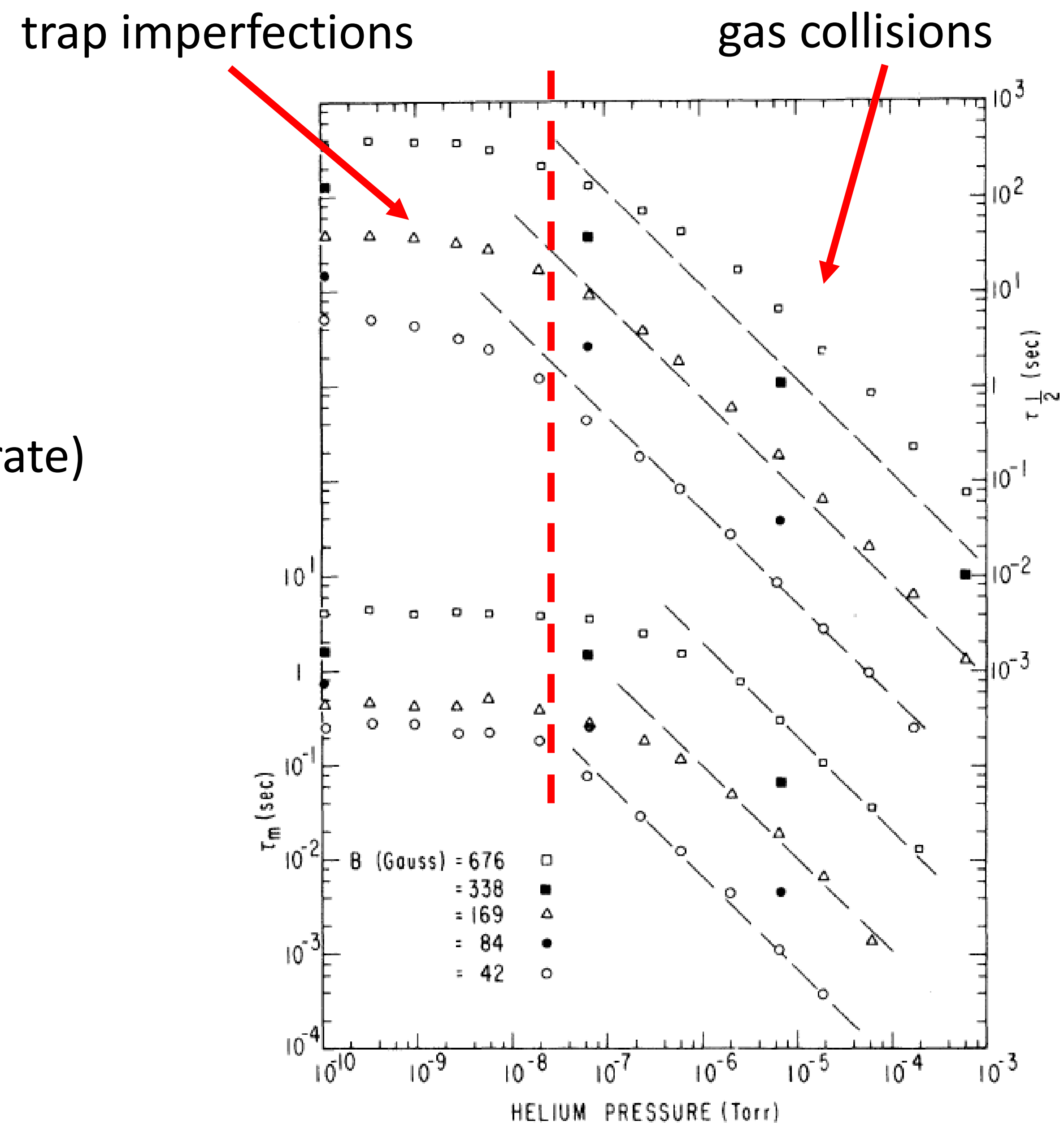
- In practice, plasmas expand slowly due to imperfections in the trap

$$\tau = \frac{dP_\theta}{dt} = P_\theta \Gamma_\theta \qquad \Gamma_\theta = \frac{1}{n}\frac{dn}{dt}$$ (outward transport rate)

- Neutral collisions

$$\Gamma_0 = \nu_p \left(\frac{r_c}{\lambda_D}\right)^2$$

ν_p - electron-neutral collision freq.



doi: 10.1103/PhysRevLett.44.654

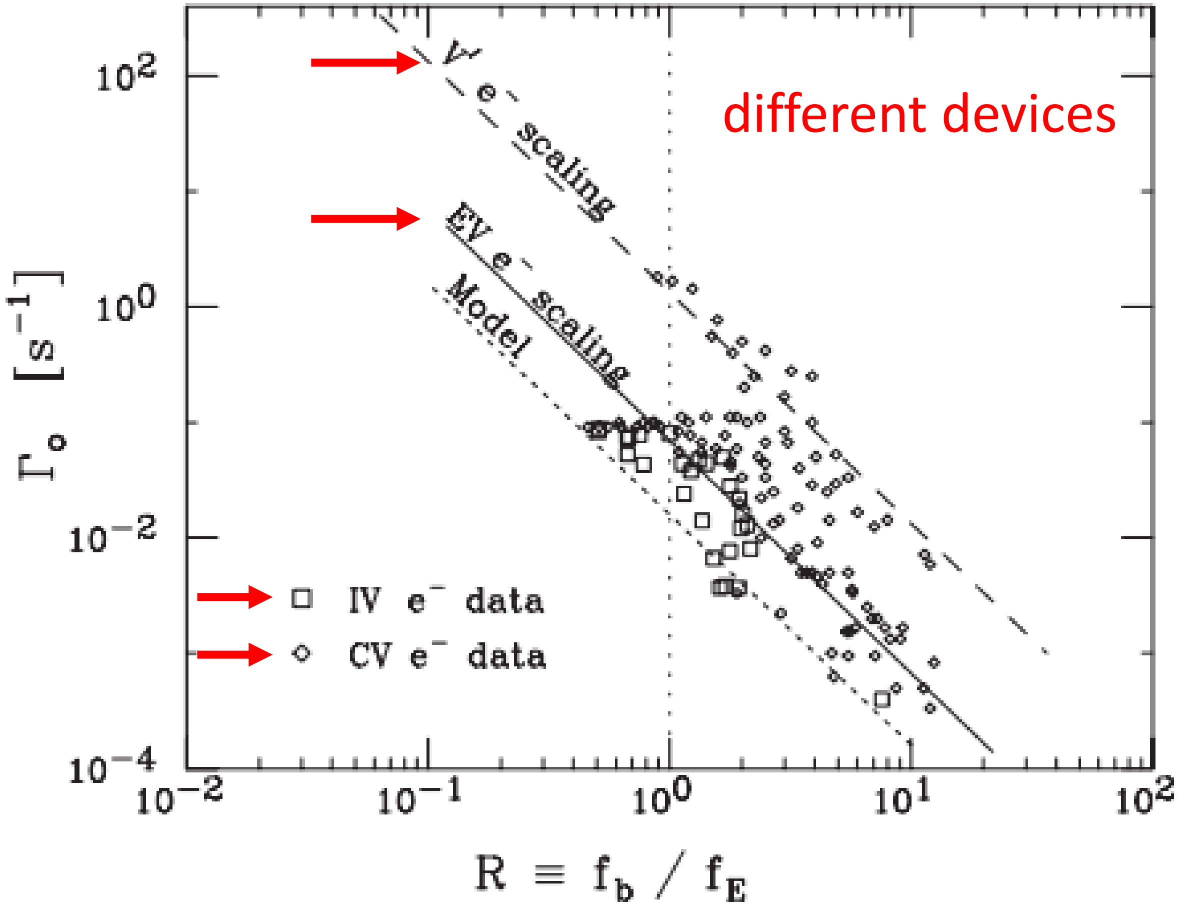
Cross-field transport

(Plasmas at low neutral pressure)

- Expansion rate

$$\Gamma_0 = \Lambda\, R^2$$

device dependent constant



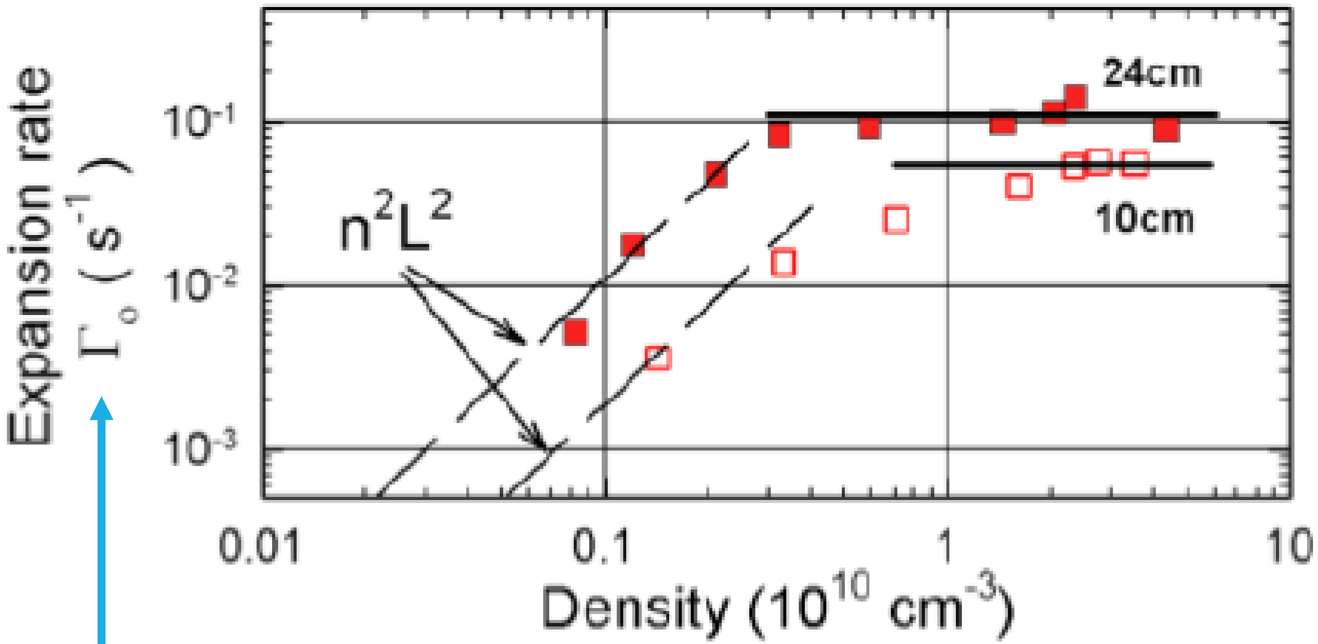
plasma “rigidity” $R = \frac{f_b}{f_E} = 1.5\left(\frac{B}{1\ \mathrm{T}}\right)\left(\frac{n}{10^{10}\ \mathrm{cm}^{-3}}\right)^{-1}\left(\frac{L_p}{1\ \mathrm{cm}}\right)^{-1}\left(\frac{T_p}{1\ \mathrm{eV}}\right)^{1/2}$

https://doi.org/10.1016/S0969-806X(03)00194-4

Cross-field transport

- Transport due to electric and magnetic asymmetries

UHV PM trap (5-tesla field)



(among the smallest values reported)

doi: 10.1063/1.2179410

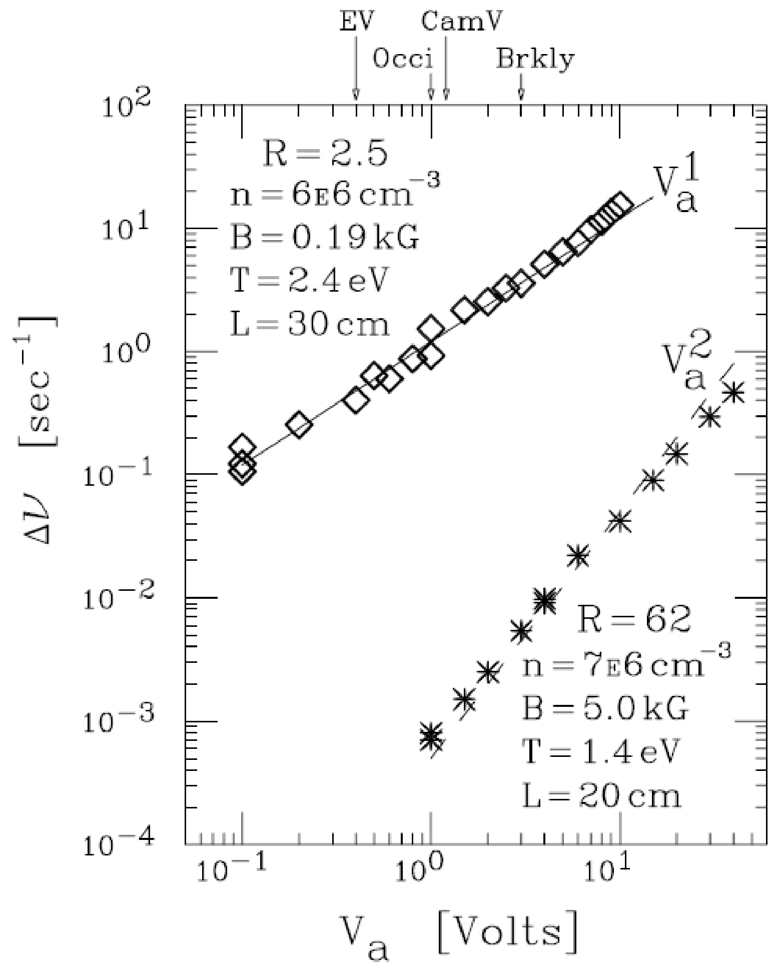


FIG. 3. Net expansion rate $\Delta\nu$ vs asymmetry strength V_a for a low rigidity plasma ($\mathcal{R}$ = 2.5) and a high rigidity plasma ($\mathcal{R}$ = 62).

doi: 10.1103/PhysRevLett.85.2510

Plasma heating and cooling

Heating sources:

- RF noise on the confining electrodes
- Outward plasma expansion itself

$$\Gamma_h = \frac{1}{T}\frac{dT}{dt} = \left(\frac{e\varphi_0}{2\eta T}\right)\Gamma_0$$

φ_0 - potential at $r = r_w$
$1/\eta$ – fraction of space-charge potential that is dropped across the plasma

https://doi.org/10.1016/S0969-806X(03)00194-4
https://doi.org/10.1142/9781783264063_0004

Cooling

- cyclotron cooling (requires $2r_w > \lambda_c$)

$$\Gamma_c = \frac{1}{4}\left(\frac{B}{1\,T}\right)^2 \ (s^{-1})$$

- collisional cooling on atomic or molecular gases

$$\Gamma_c = \frac{1}{T}\frac{dT}{dt} \approx -\frac{\nu_j \varepsilon_j}{T}$$

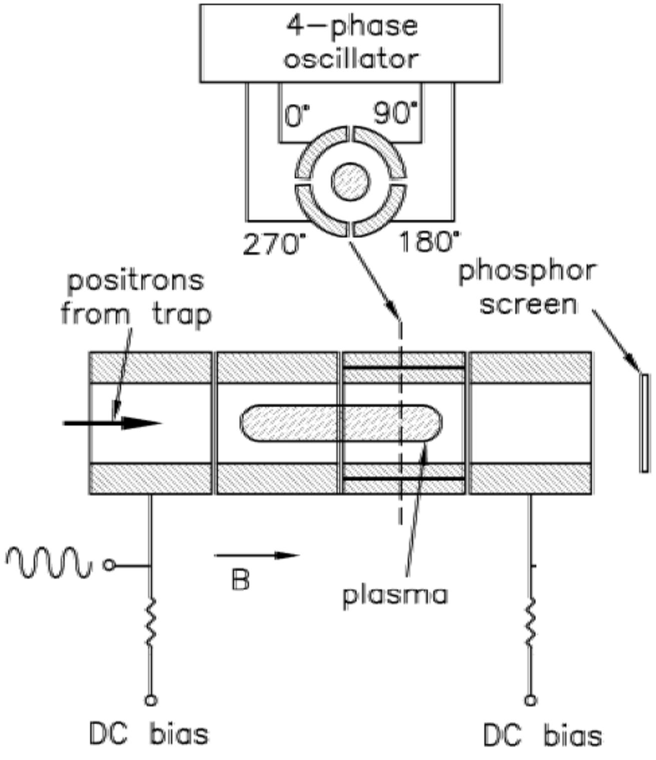
ν_j - excitation rate for a vibrational mode j
ε_j - transition energy for mode j

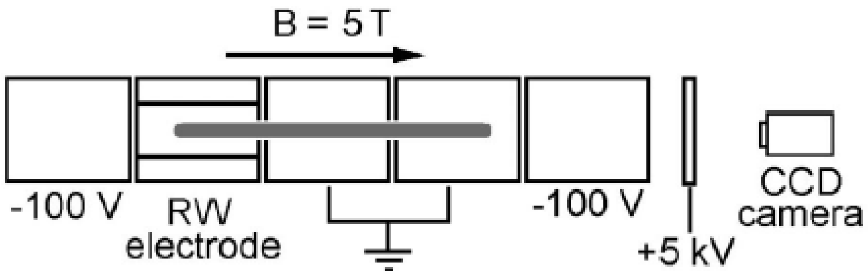
- If neutral collisions dominate both the transport and the cooling:

$$\beta = \frac{\Gamma_h}{\Gamma_c} = \frac{\nu_p T}{4\nu_j \varepsilon_j}\left(\frac{\omega_p}{\omega_c}\right)^2\left(\frac{r_p}{\lambda_D}\right)^2$$

$$\boxed{\left(\frac{n}{n_B}\right)^2 = \frac{8\nu_j \varepsilon_j}{\nu_p T}\left(\frac{r_c}{r_p}\right)^2}$$

Rotating wall technique





Rotating electric field injects angular momentum

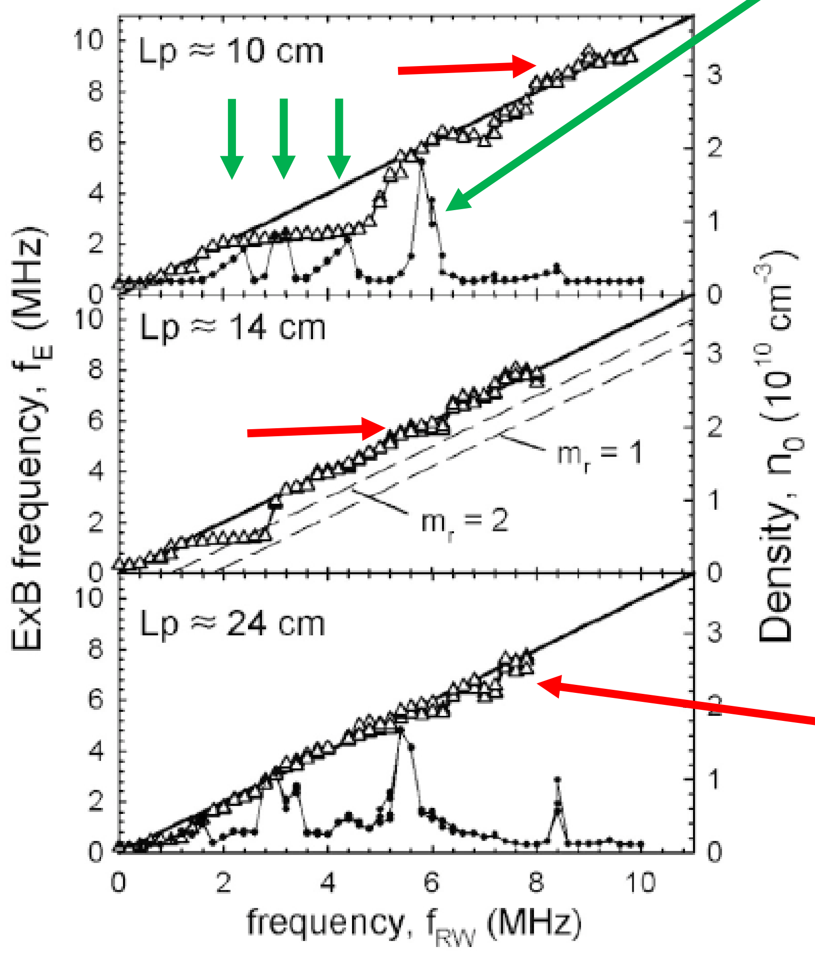
- Good compression for the axial extent of the RW electrode less than half L_p
- Usually RW electrode located at one end
- Radial electric field rotating in the same direction as the plasma

$$P_\theta \cong -\frac{eB}{2}\sum_j r_j^2$$

$$\boxed{\begin{array}{ll} f_{RW} > f_E & \text{compression} \\ f_{RW} < f_E & \text{expansion} \end{array}}$$

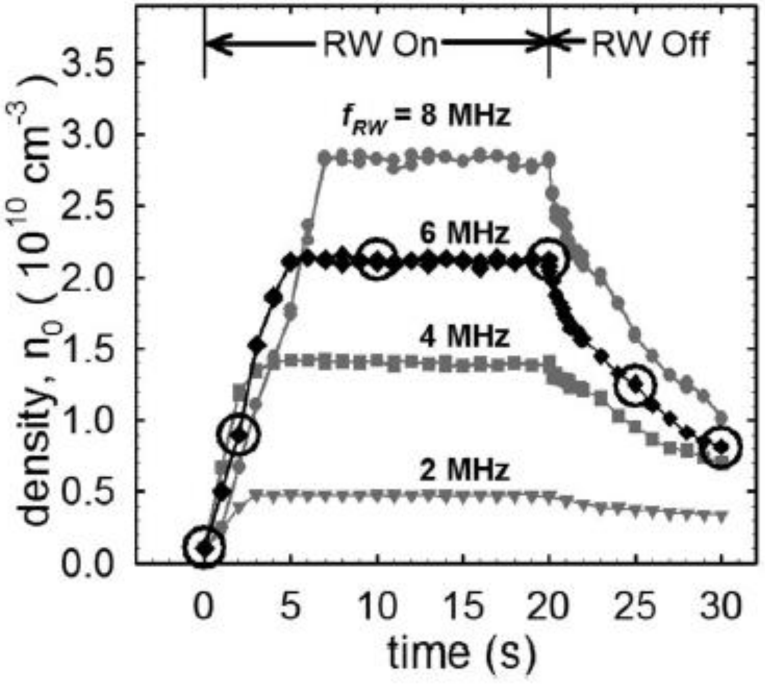
- Cooling is required to counteract the heating caused by the torque-produced work on the plasma

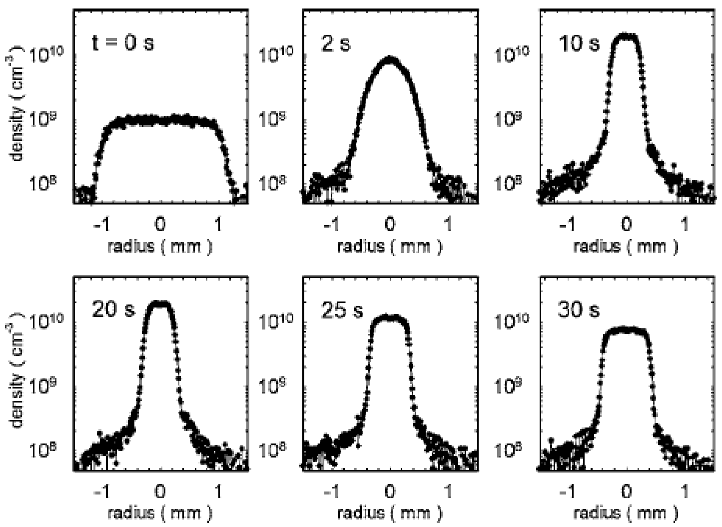
Rotating wall technique



Early RW experiments: coupling to Trivelpiece-Gould modes ($V_{RW} = 0.1\ V$)

"strong-drive" regime ($V_{RW} = 1.0\ V$)

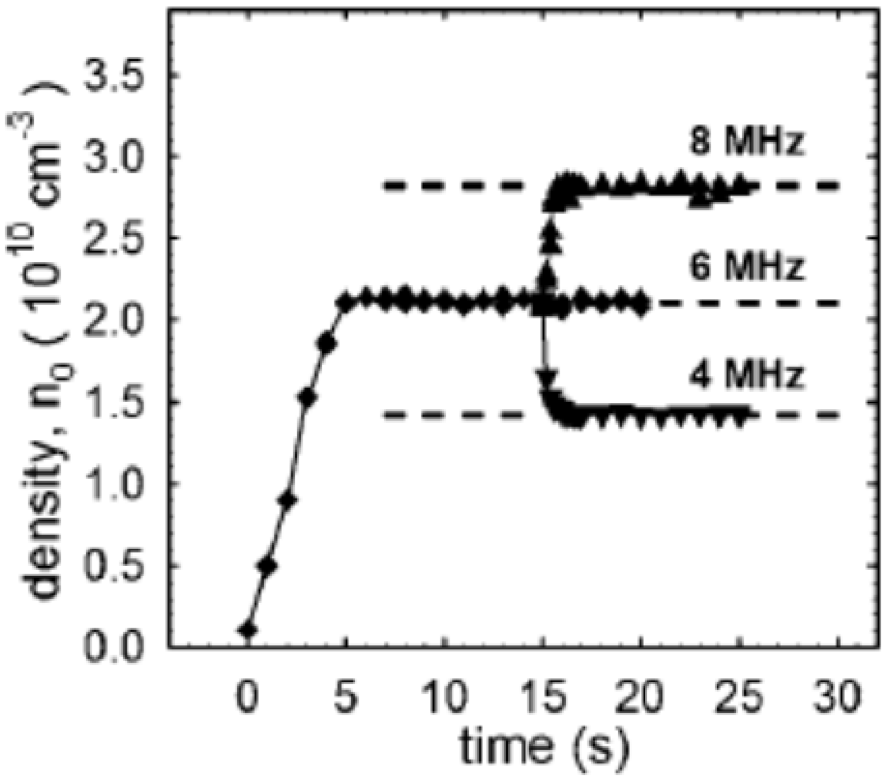




doi: 10.1063/1.2179410
doi: 10.1103/PhysRevLett.94.035001

Rotating wall technique

"One-knob density control"



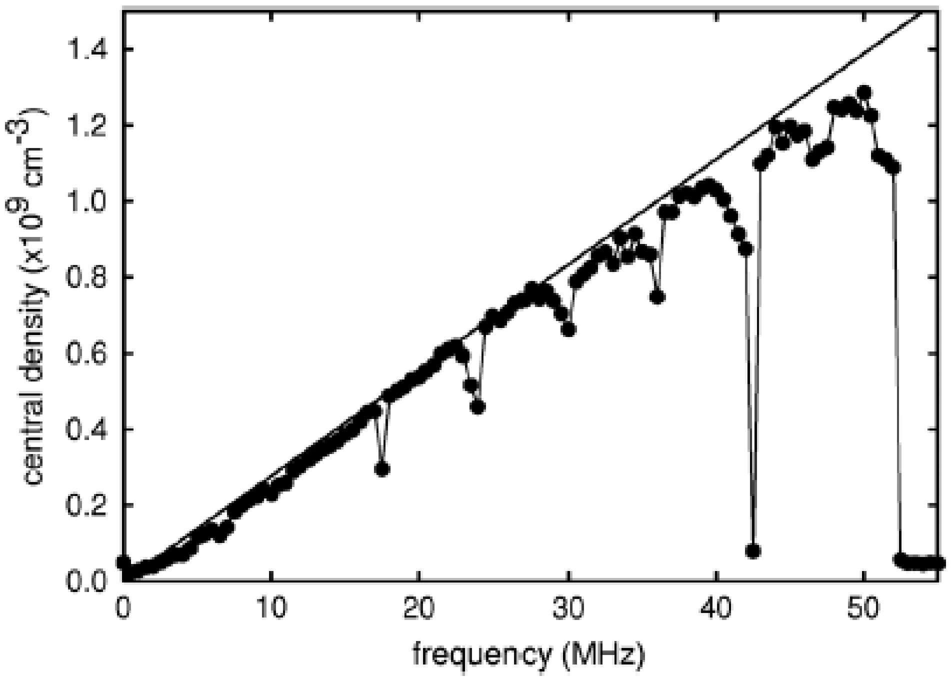
[doi: 10.1063/1.2179410]

Approaching the Brillouin limit

Record so far
(R.G. Greaves, unpublished):

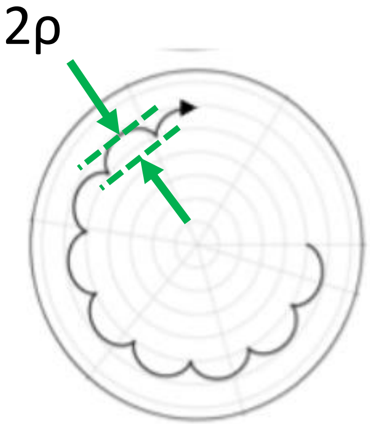
$\mathbf{1.2 \times 10^{15}\ m^{-3}}$ $(\mathbf{17\%}\ \text{of}\ n_B)$

$B \sim 0.04\ T \qquad f_{RW} \sim 50\ MHz$



High density => large space-charge, large non-circular orbits
Higher density limited by molecular collisions

nearly
unconfined
$E \times B$ orbits

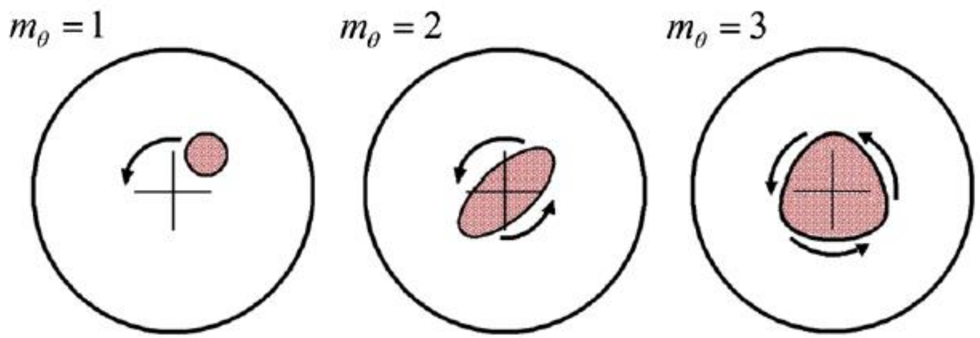


[doi: 10.1103/RevModPhys.87.247]
[doi: 10.1063/1.3354005]
[doi: 10.1142/9781783264063_0005]

$$\rho = \sqrt{r_c^2 + r_E^2} \qquad \rho \approx r_E = E/\omega_c B >> r_c$$

Non-destructive diagnostic techniques

Diocotron modes ($k_z = 0$ plasma oscillations)



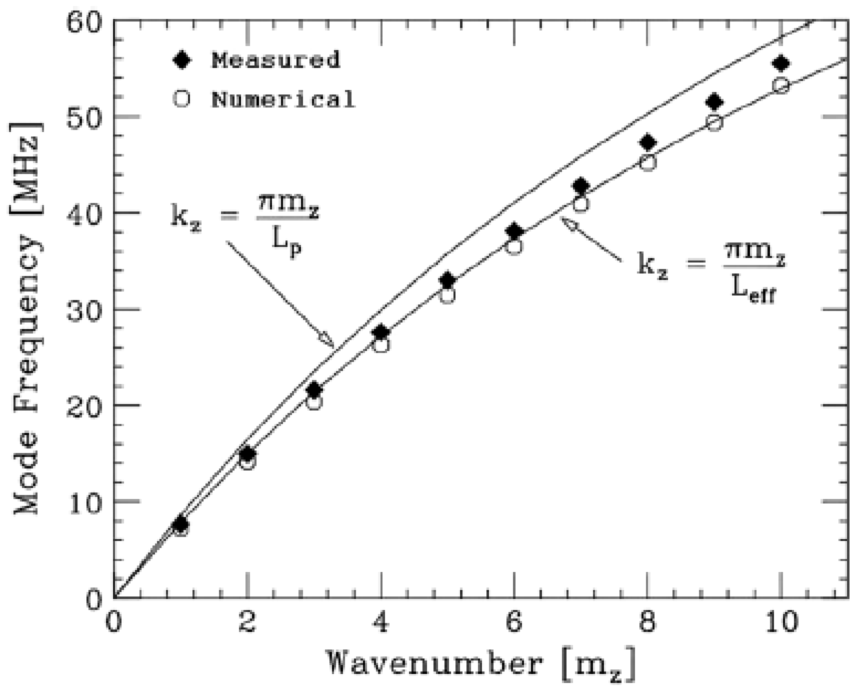
$$\omega_1 = -\frac{N}{L_p}\frac{qc}{2\pi\varepsilon_0 B r_w^2} \longrightarrow N$$

$$\omega_2 \cong -\omega_r \longrightarrow n$$

[doi: 10.1103/RevModPhys.87.247]

Trivelpiece-Gould modes (finite k_z plasma oscillations)

$$\omega_{TG} \approx \frac{k_z}{k_\perp}\omega_p\left[1 + \frac{3}{2}\lambda_D^2 k_\perp^2\right] \longrightarrow T$$



Summary

- Plasma can reach steady state – rigid rotor
- Strongly magnetized plasma => particles "tied" to the field lines
- Diagnostics: oscillation modes to measure N, n, T and aspect ratio
- Outward transport (trap imperfections, gas collisions)
- "Anomalous transport" in UHV
- Expansion => Heating
- Compression possible with RW
  - Minimise transport
  - Maximize cooling
- Density limits yet to be determined