LhARA: Capture Meeting

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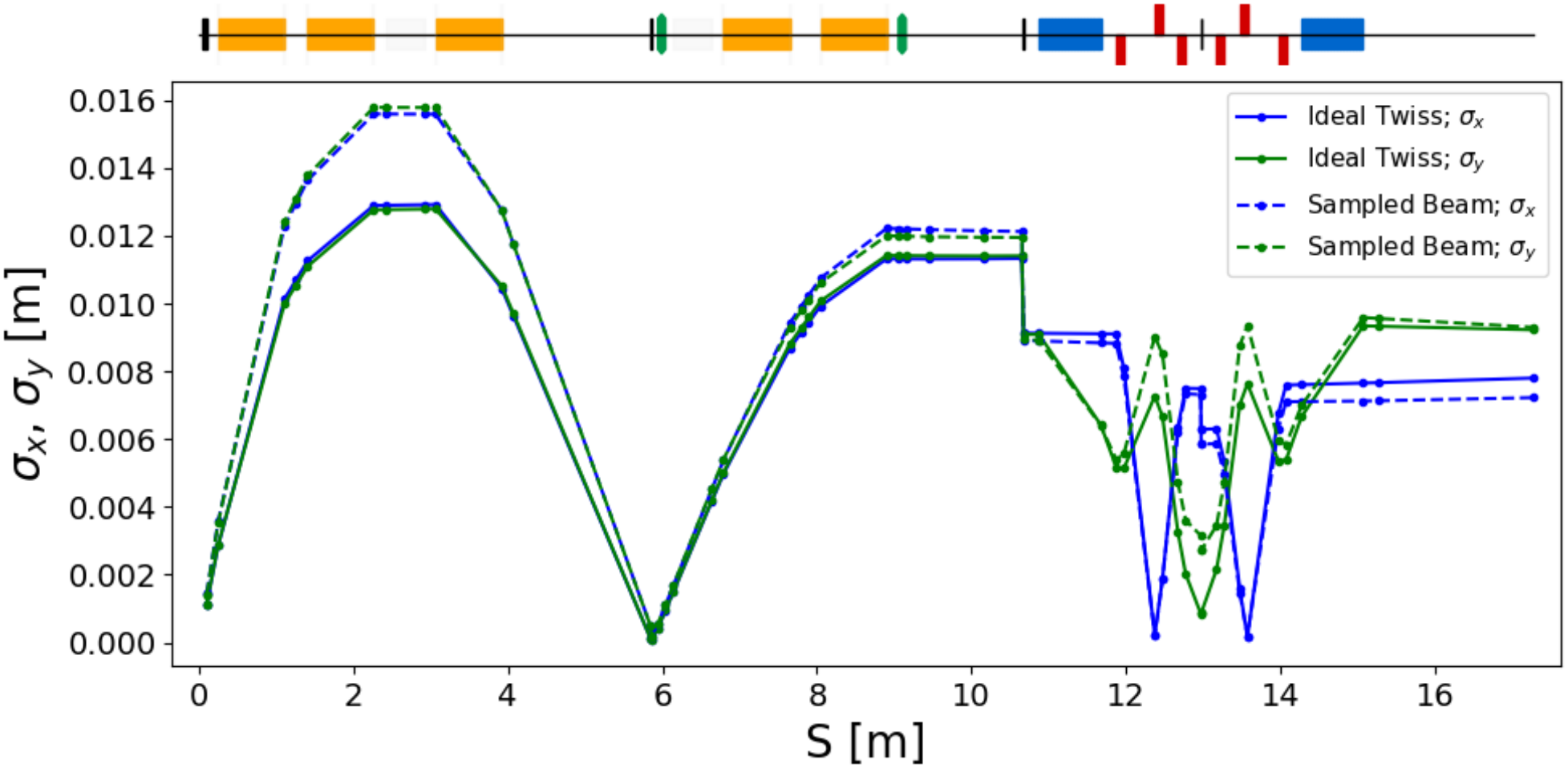
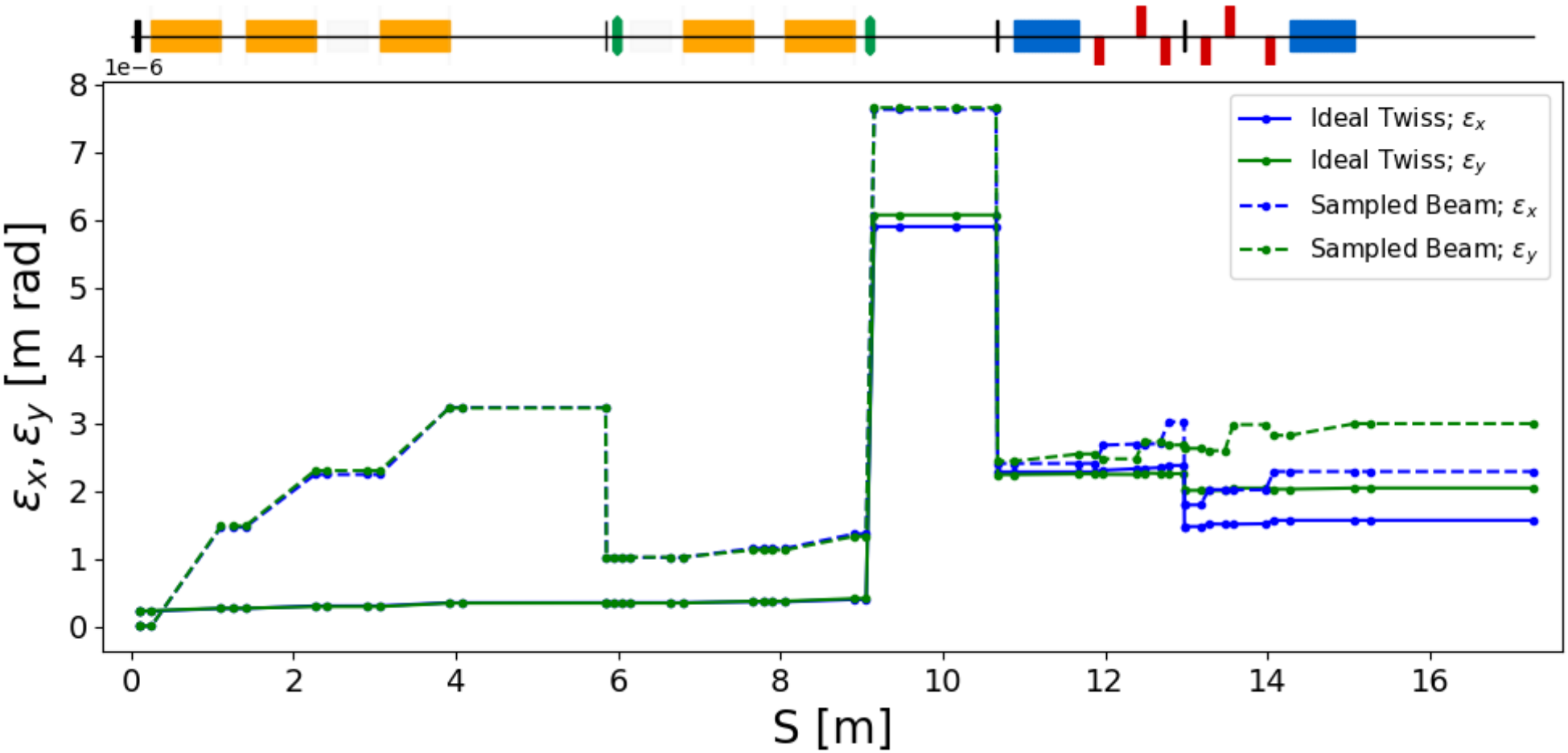
May 6, 2021

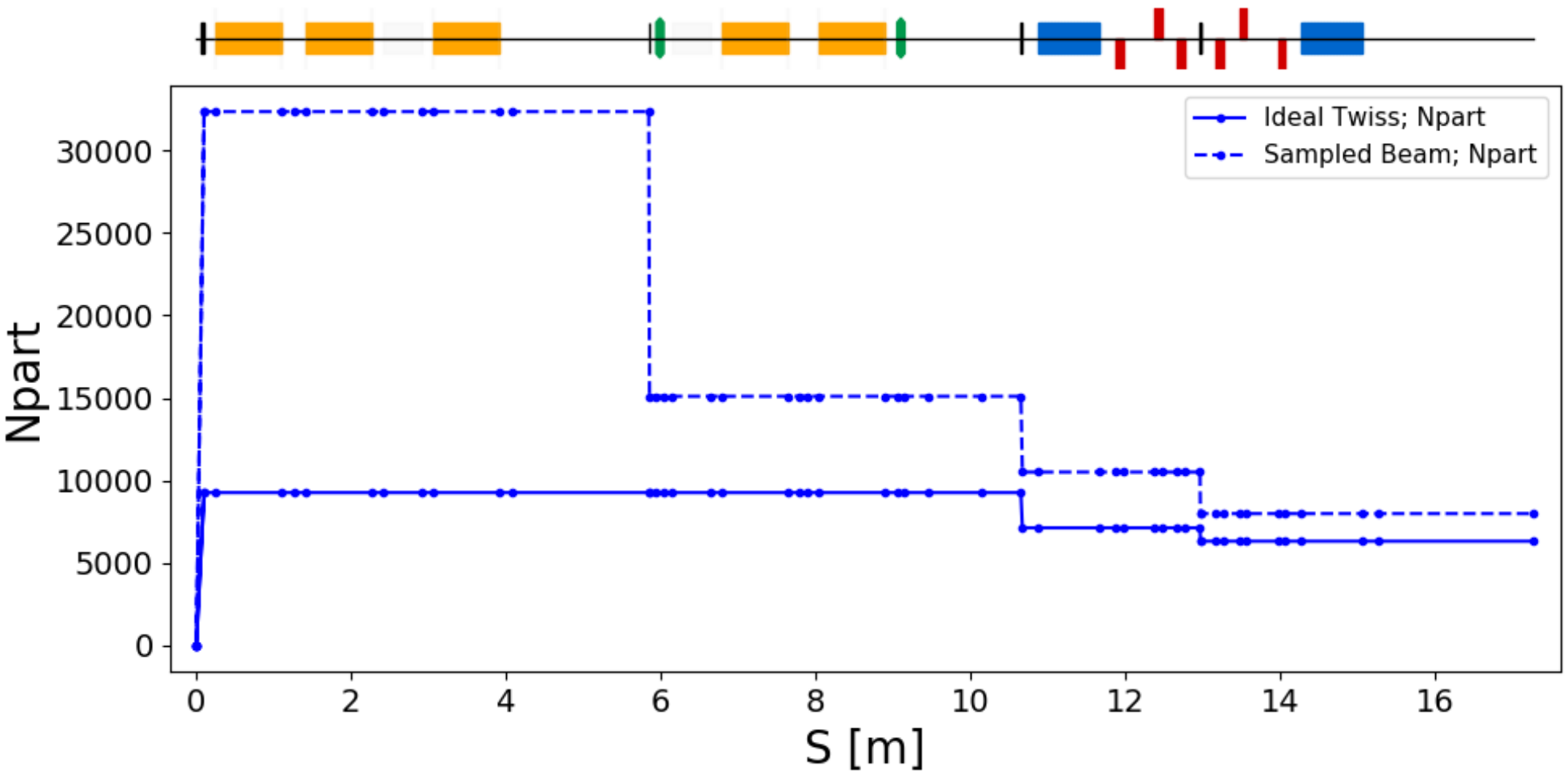
Beam Size Evolution Comparison against Ideal Twiss Beam



Beam Emittance Evolution Comparison against Ideal Twiss Beam



Beam Losses


Ideal Twiss; Npart
Sampled Beam; Npart
Npart
30000
25000
20000
15000
10000
5000
0
0
2
4
6
8
10
12
14
16
S [m]

Checking Numerical Operations

Small emittance is potentially an issue with order of magnitude of mathematical operations.
Calculating an element of covariance matrix:

$$\sigma_{x,x'} = \frac{1}{N}\sum_{i=0}^{N}(x_i - \overline{x})(x_i' - \overline{x'}) \tag{1}$$

$$= \frac{1}{N}\sum_{i=0}^{N}\left(x_i x_i' - x_i\overline{x'} - \overline{x}x_i' + \overline{x}\,\overline{x'}\right)$$

$$= \frac{1}{N}\left(\sum_{i=0}^{N}\left(x_i x_i'\right) - N\overline{x}\,\overline{x'} - N\overline{x}\,\overline{x'} + N\overline{x}\,\overline{x'}\right)$$

$$= \frac{1}{N}\left(\sum_{i=0}^{N} x_i x_i'\right) - \overline{x}\,\overline{x'} \tag{2}$$

Quoting some numbers for sampled beam:

$$\sigma_{x,x'} = (2.05526e-05) - (7.08915e-09)$$

Checking Numerical Operations

However, going through the two methods only gives a minor discrepancy in emittance ($\varepsilon_x = \sqrt{\begin{vmatrix} \sigma_{x,x} & \sigma_{x,x'} \\ \sigma_{x',x} & \sigma_{x',x'} \end{vmatrix}}$):

Ideal Beam:

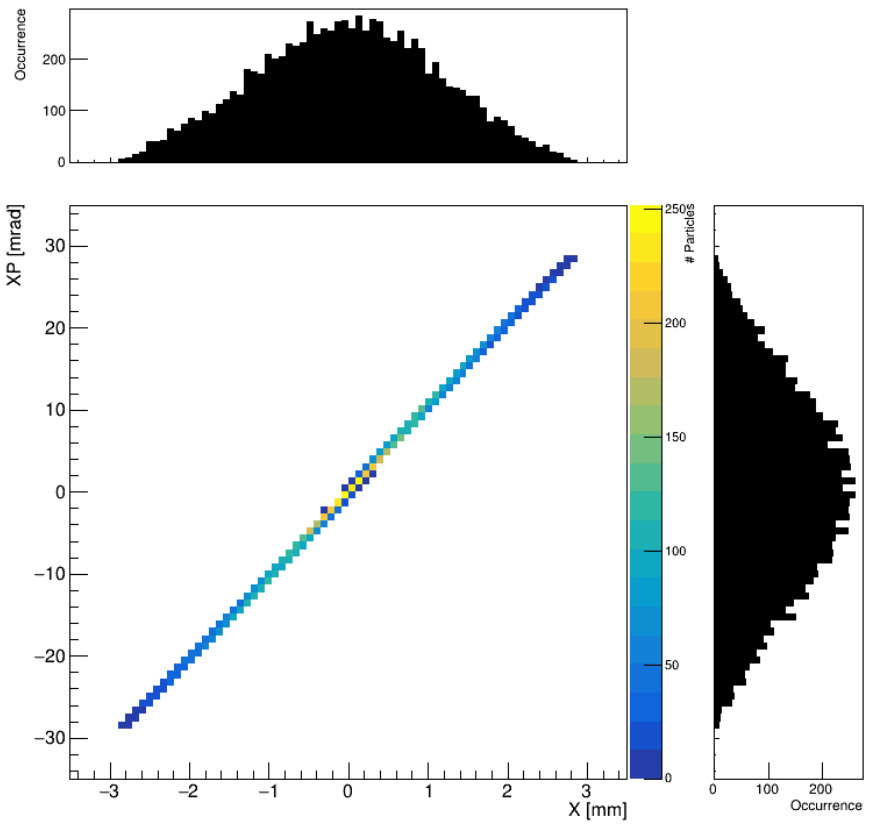
$$\varepsilon_{x,\text{1st Method}} = 2.324388578e-07 \qquad \varepsilon_{y,\text{1st Method}} = 2.397319415e-07$$
$$\varepsilon_{x,\text{2nd Method}} = 2.324388898e-07 \qquad \varepsilon_{y,\text{2nd Method}} = 2.397320708e-07$$
$$\text{Percent Error } \varepsilon_x = (1.38e-05)\% \qquad \text{Percent Error } \varepsilon_y = (5.39e-05)\%$$

Sampled Beam:

$$\varepsilon_{x,\text{1st Method}} = 1.474804831e-08 \qquad \varepsilon_{y,\text{1st Method}} = 1.406784704e-08$$
$$\varepsilon_{x,\text{2nd Method}} = 1.474893357e-08 \qquad \varepsilon_{y,\text{2nd Method}} = 1.407470219e-08$$
$$\text{Percent Error } \varepsilon_x = 0.006\% \qquad \text{Percent Error } \varepsilon_y = 0.049\%$$

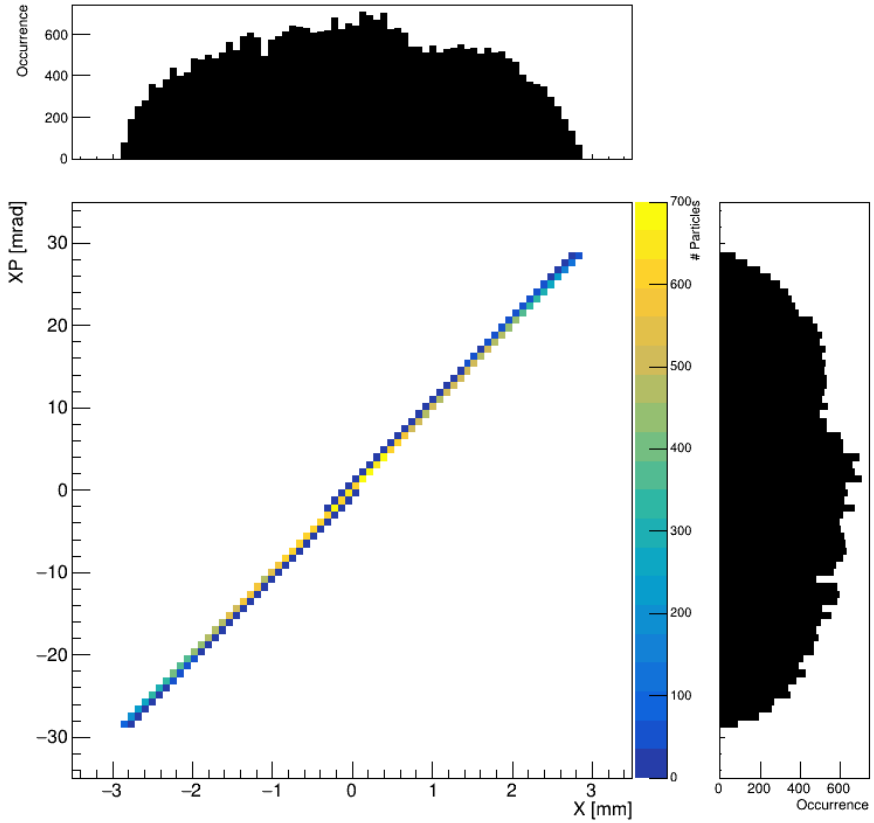
Exit to Nozzle (S = 0.1 m)

Ideal Beam: Nozzle End



β_x [m]	5.4 ± 0.1
α_x	-56.0 ± 0.4
ϵ_x [m rad]	$2.3e-07 \pm (2.6e-09)$
β_y [m]	5.3 ± 0.1
α_y	-55.2 ± 0.4
ϵ_y [m rad]	$2.4e-07 \pm (2.7e-09)$

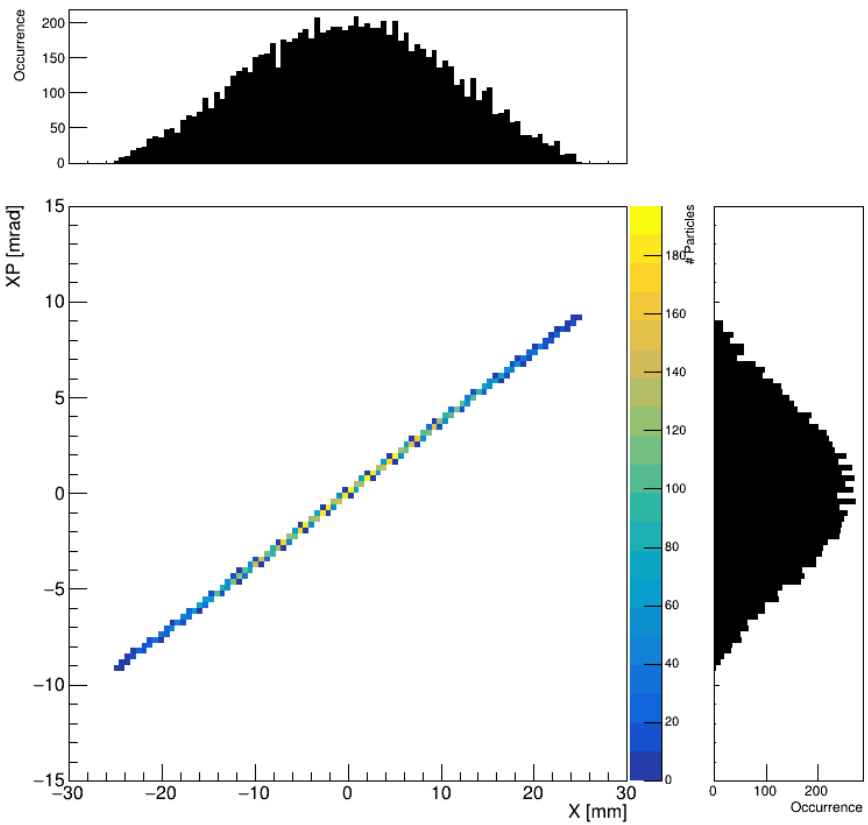
Sampled Beam: Nozzle End



β_x [m]	145.4 ± 0.7
α_x	-1458.6 ± 6.8
ϵ_x [m rad]	$1.4e-08 \pm (8.0e-11)$
β_y [m]	149.1 ± 0.8
α_y	-1496.3 ± 8.4
ϵ_y [m rad]	$1.3e-08 \pm (8.3e-11)$

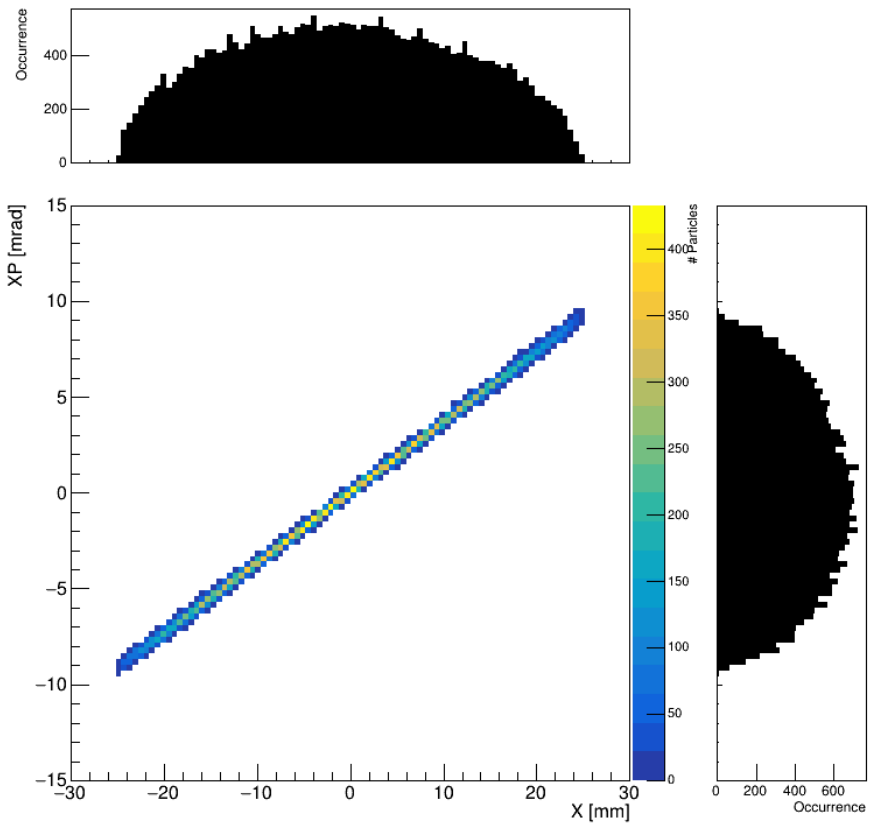
Exit to Gabor Lens 1 (S = 1.07 m)

Ideal Beam: Gabor Lens 1 End



β_x [m]	374.83 ± 2.73
α_x	-139.27 ± 1.01
ϵ_x [m rad]	$2.74e-07 \pm (3.04e-09)$
β_y [m]	359.4 ± 2.74
α_y	-133.58 ± 1.01
ϵ_y [m rad]	$2.77e-07 \pm (3.22e-09)$

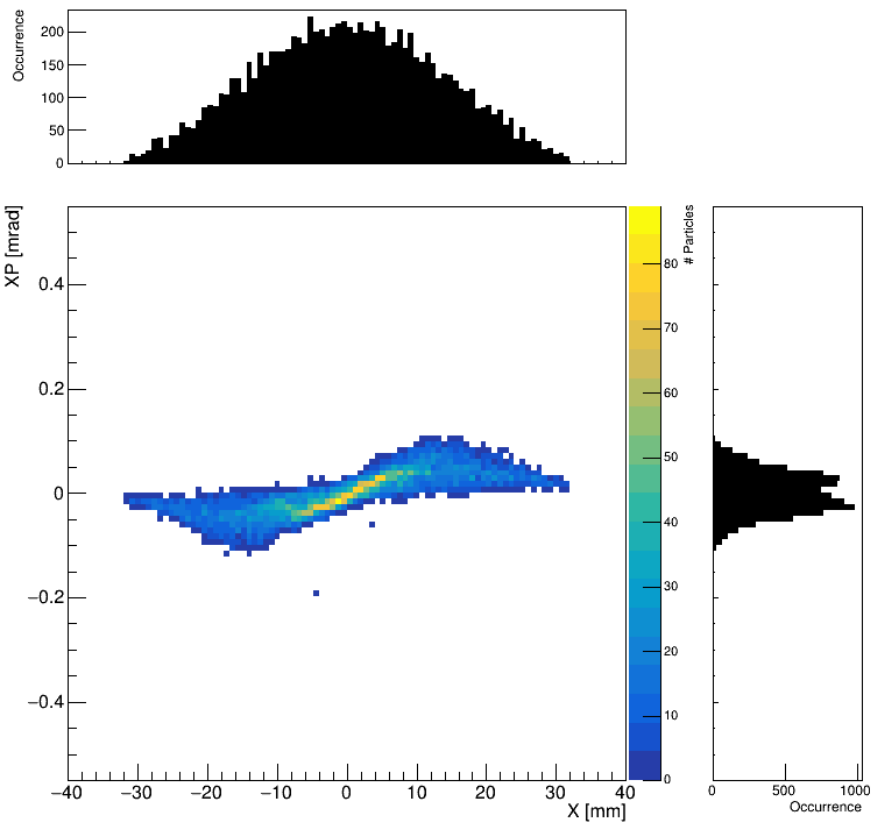
Sampled Beam: Gabor Lens 1 End



β_x [m]	102.47 ± 0.47
α_x	-37.62 ± 0.17
ϵ_x [m rad]	$1.47e-06 \pm (8.65e-09)$
β_y [m]	103.44 ± 0.48
α_y	-37.97 ± 0.18
ϵ_y [m rad]	$1.49e-06 \pm (8.78e-09)$

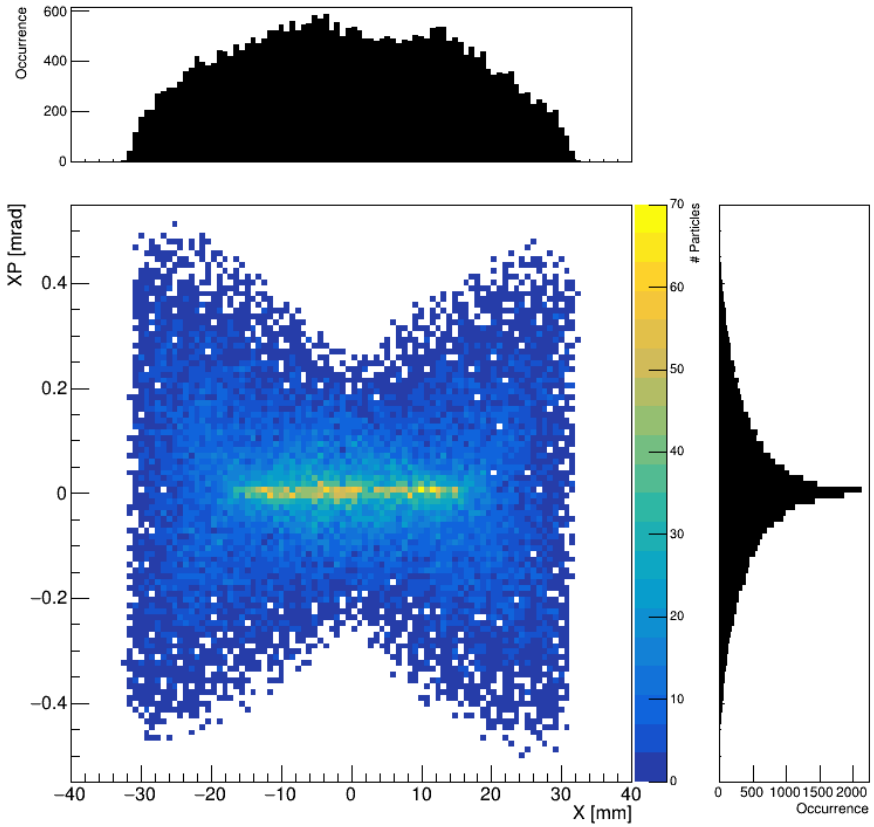
Exit to Gabor Lens 2 (S = 2.264 m)

Ideal Beam: Gabor Lens 2 End



β_x [m]	533.59 ± 4.46
α_x	-1.28 ± 0.01
ϵ_x [m rad]	$3.12e-07 \pm (3.55e-09)$
β_y [m]	538.73 ± 4.51
α_y	-1.32 ± 0.01
ϵ_y [m rad]	$3.02e-07 \pm (3.46e-09)$

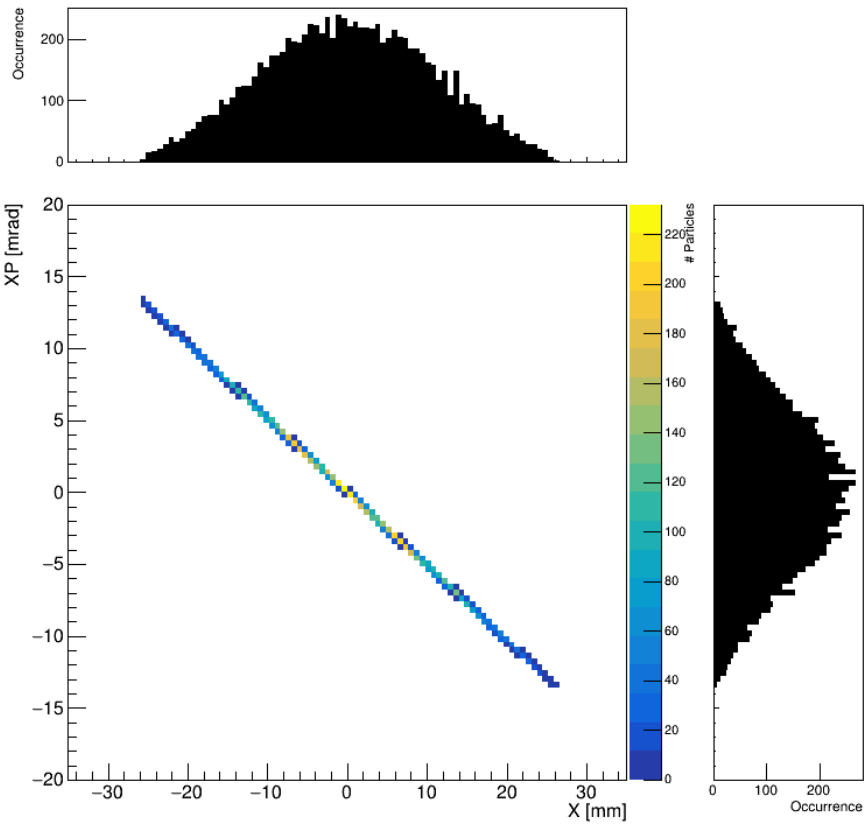
Sampled Beam: Gabor Lens 2 End



β_x [m]	108.13 ± 0.46
α_x	0.06 ± 0.01
ϵ_x [m rad]	$2.25e-06 \pm (1.39e-08)$
β_y [m]	108.06 ± 0.46
α_y	0.05 ± 0.01
ϵ_y [m rad]	$2.31e-06 \pm (1.42e-08)$

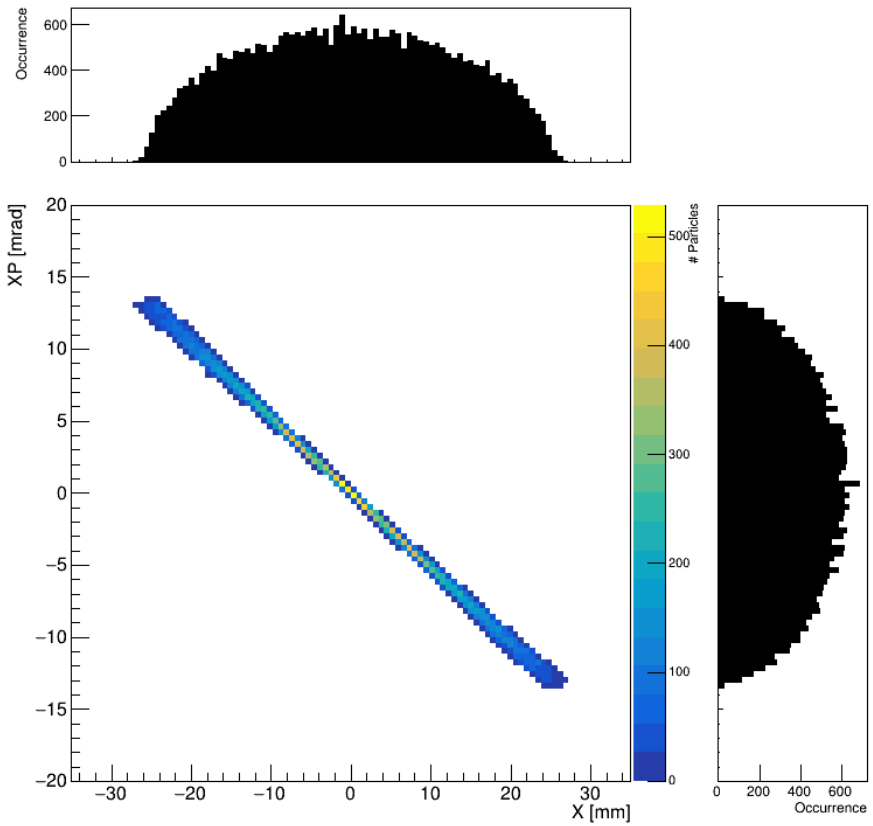
Exit to Gabor Lens 3 (S = 3.921 m)

Ideal Beam: Gabor Lens 3 End



β_x [m]	303.78 ± 2.8
α_x	156.24 ± 1.45
ϵ_x [m rad]	$3.59e-07 \pm (4.34e-09)$
β_y [m]	313.04 ± 2.78
α_y	161.03 ± 1.44
ϵ_y [m rad]	$3.53e-07 \pm (4.10e-09)$

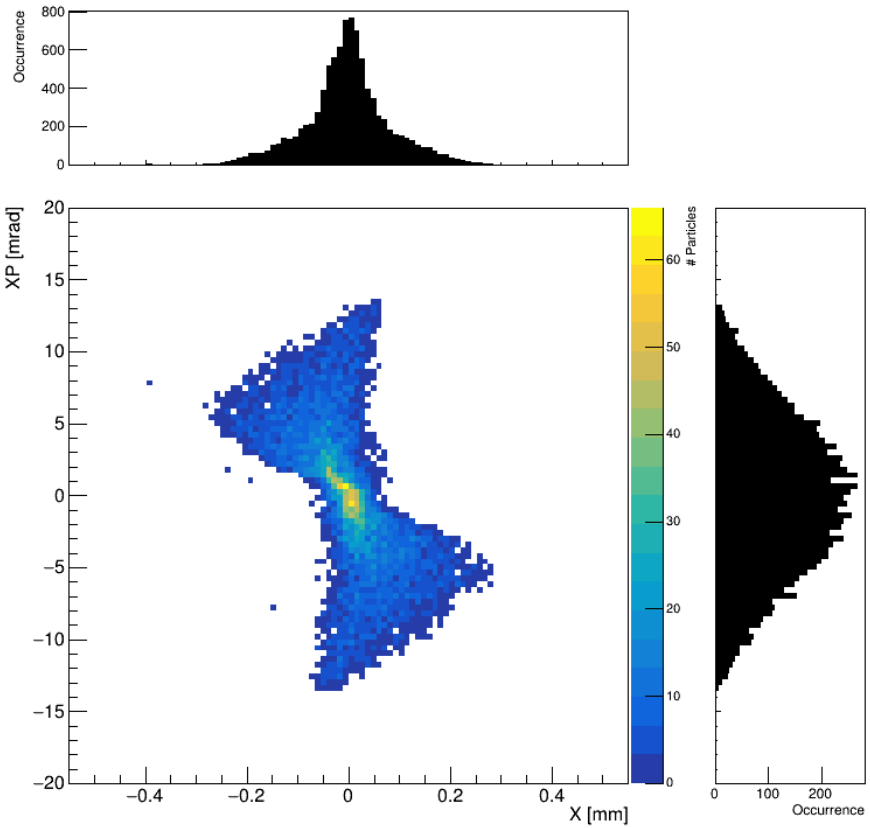
Sampled Beam: Gabor Lens 3 End



β_x [m]	50.24 ± 0.19
α_x	26.06 ± 0.1
ϵ_x [m rad]	$3.24e-06 \pm (2.11e-08)$
β_y [m]	50.34 ± 0.19
α_y	26.11 ± 0.1
ϵ_y [m rad]	$3.23e-06 \pm (2.09e-08)$

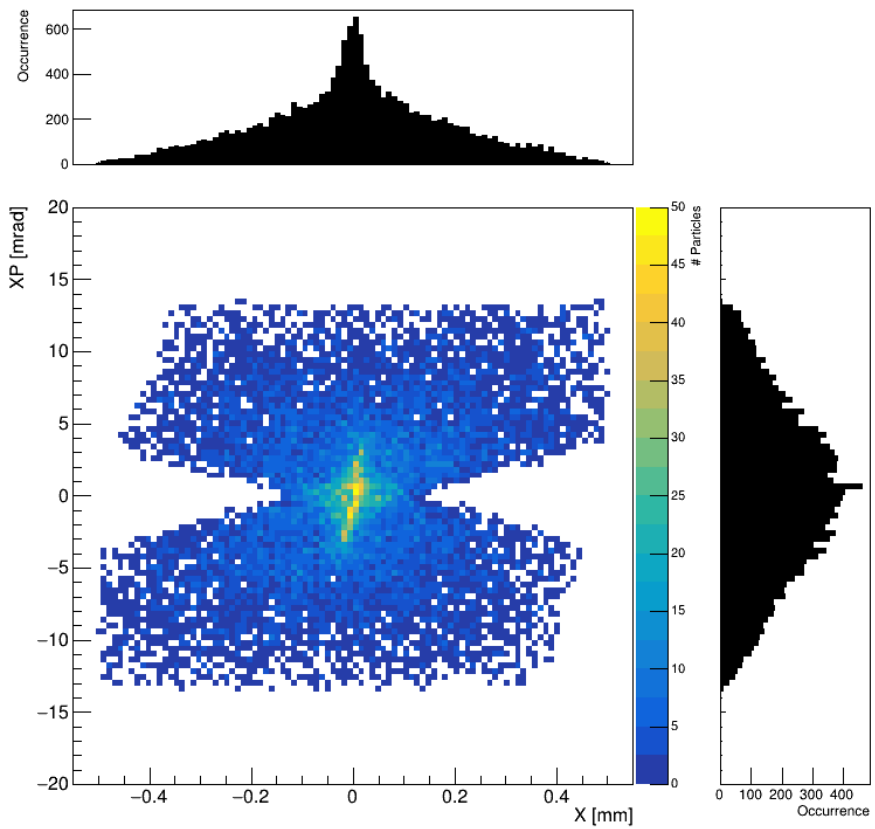
Exit to Collimator 1 (S = 5.765 m) – need to verify

Ideal Beam: Collimator 1 End



β_x [m]	0.02 ± 0.0
α_x	0.7 ± 0.01
ϵ_x [m rad]	$3.59e-07 \pm (4.23e-09)$
β_y [m]	0.02 ± 0.0
α_y	0.7 ± 0.01
ϵ_y [m rad]	$3.53e-07 \pm (4.08e-09)$

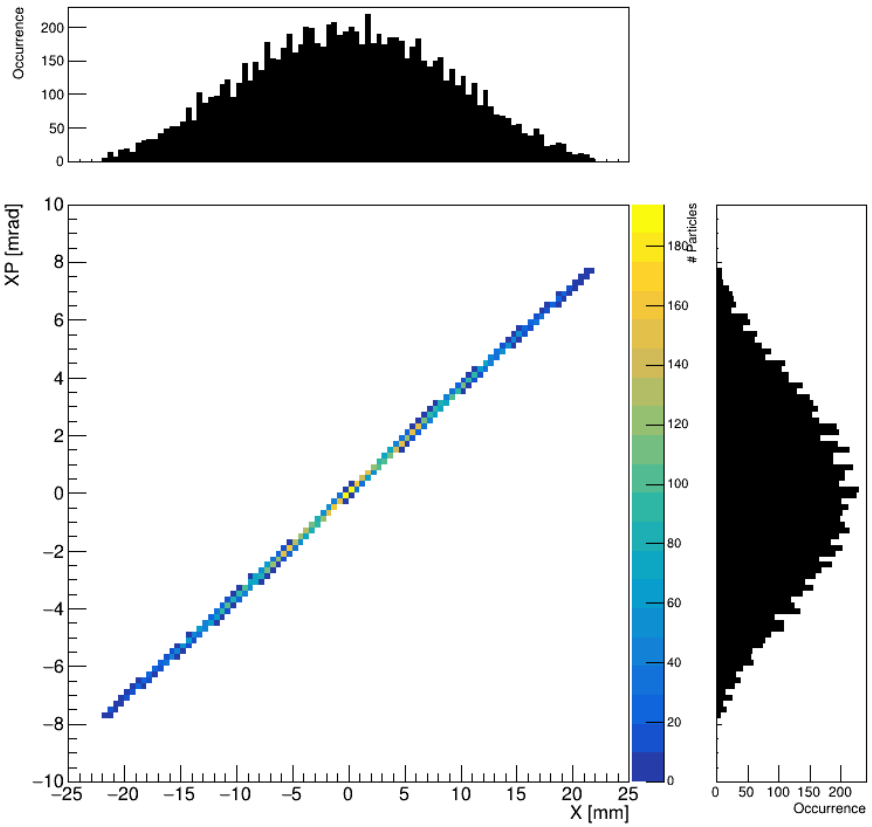
Sampled Beam: Collimator 1 End



β_x [m]	0.03 ± 0.0
α_x	-0.2 ± 0.01
ϵ_x [m rad]	$1.03e-06 \pm (8.63e-09)$
β_y [m]	0.03 ± 0.0
α_y	-0.19 ± 0.01
ϵ_y [m rad]	$1.03e-06 \pm (8.70e-09)$

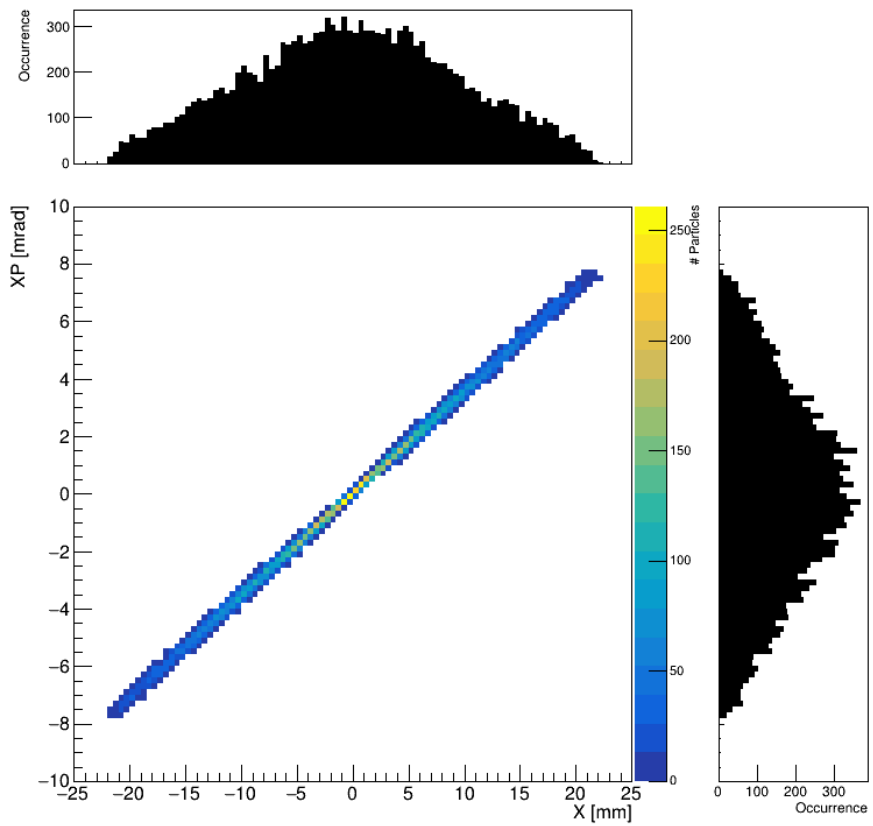
Exit to Gabor Lens 4 (S = 7.648 m) – need to verify

Ideal Beam: Gabor Lens 4 End



β_x [m]	202.05 ± 1.82
α_x	-72.88 ± 0.65
ϵ_x [m rad]	$3.72e-07 \pm (4.28e-09)$
β_y [m]	204.59 ± 1.86
α_y	-73.76 ± 0.66
ϵ_y [m rad]	$3.79e-07 \pm (4.31e-09)$

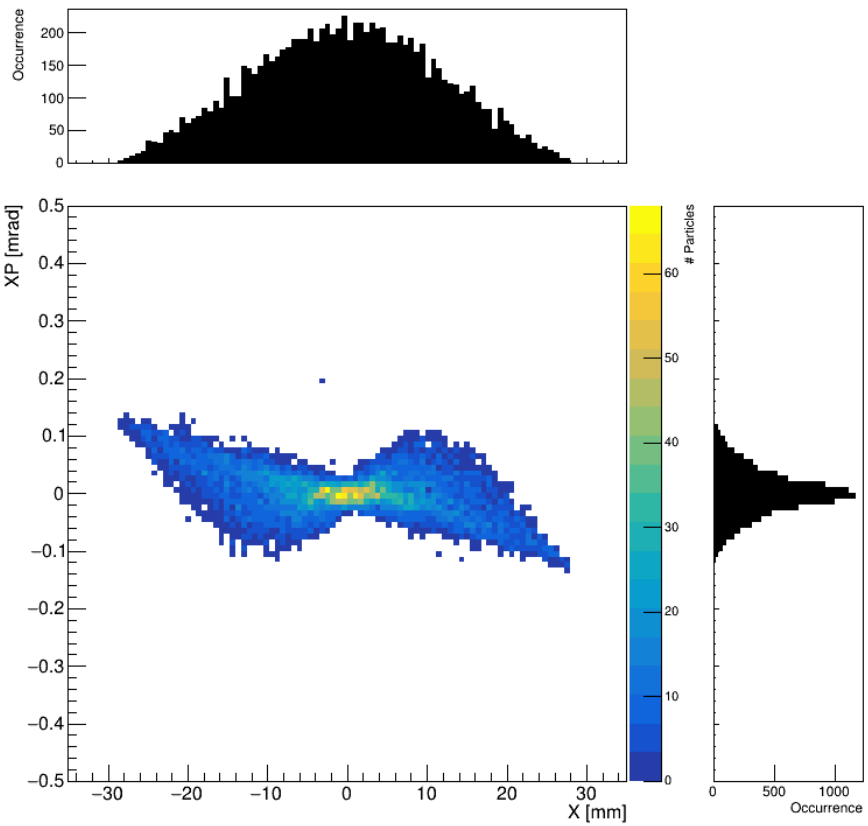
Sampled Beam: Gabor Lens 4 End



β_x [m]	76.56 ± 0.48
α_x	-27.16 ± 0.17
ϵ_x [m rad]	$1.16e-06 \pm (9.67e-09)$
β_y [m]	76.14 ± 0.48
α_y	-27.01 ± 0.17
ϵ_y [m rad]	$1.13e-06 \pm (9.44e-09)$

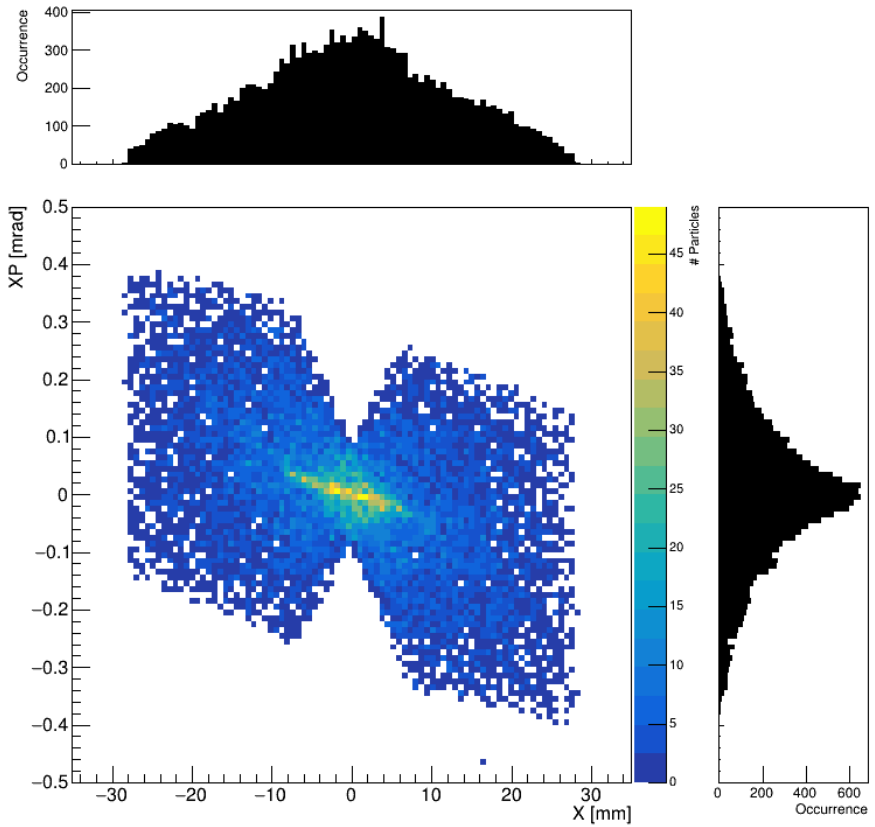
Exit to Gabor Lens 5 (S = 8.905 m) – need to verify

Ideal Beam: Gabor Lens 5 End



β_x [m]	315.17 ± 2.91
α_x	0.49 ± 0.02
ϵ_x [m rad]	$4.07e-07 \pm (4.62e-09)$
β_y [m]	310.87 ± 2.96
α_y	0.51 ± 0.02
ϵ_y [m rad]	$4.19e-07 \pm (4.84e-09)$

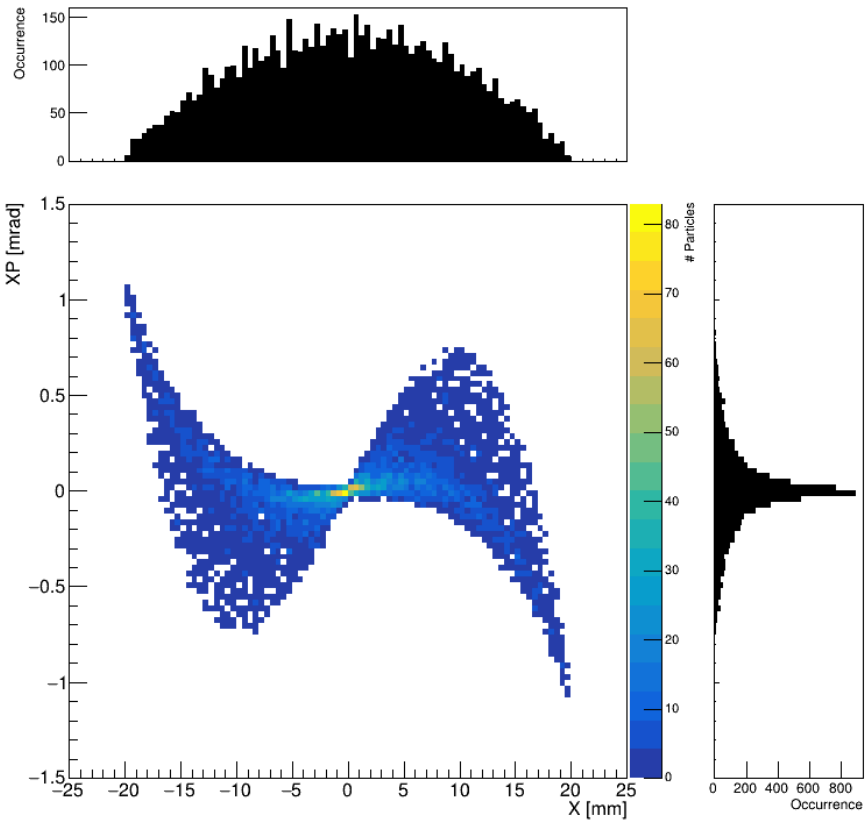
Sampled Beam: Gabor Lens 5 End



β_x [m]	108.83 ± 0.69
α_x	0.54 ± 0.01
ϵ_x [m rad]	$1.37e-06 \pm (1.15e-08)$
β_y [m]	107.67 ± 0.67
α_y	0.56 ± 0.01
ϵ_y [m rad]	$1.34e-06 \pm (1.12e-08)$

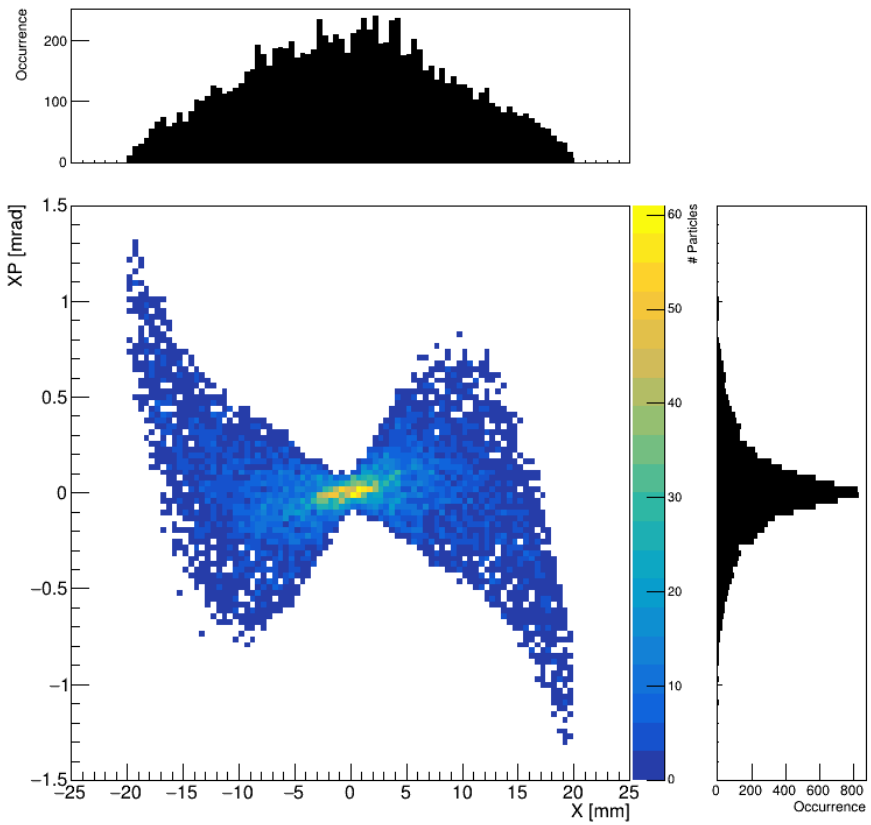
Exit to Collimator 2 (S = 10.675 m) – need to verify

Ideal Beam: Collimator 2 End



β_x [m]	36.43 ± 0.36
α_x	0.09 ± 0.02
ϵ_x [m rad]	$2.29e-06 \pm (3.55e-08)$
β_y [m]	36.8 ± 0.37
α_y	0.08 ± 0.02
ϵ_y [m rad]	$2.25e-06 \pm (3.52e-08)$

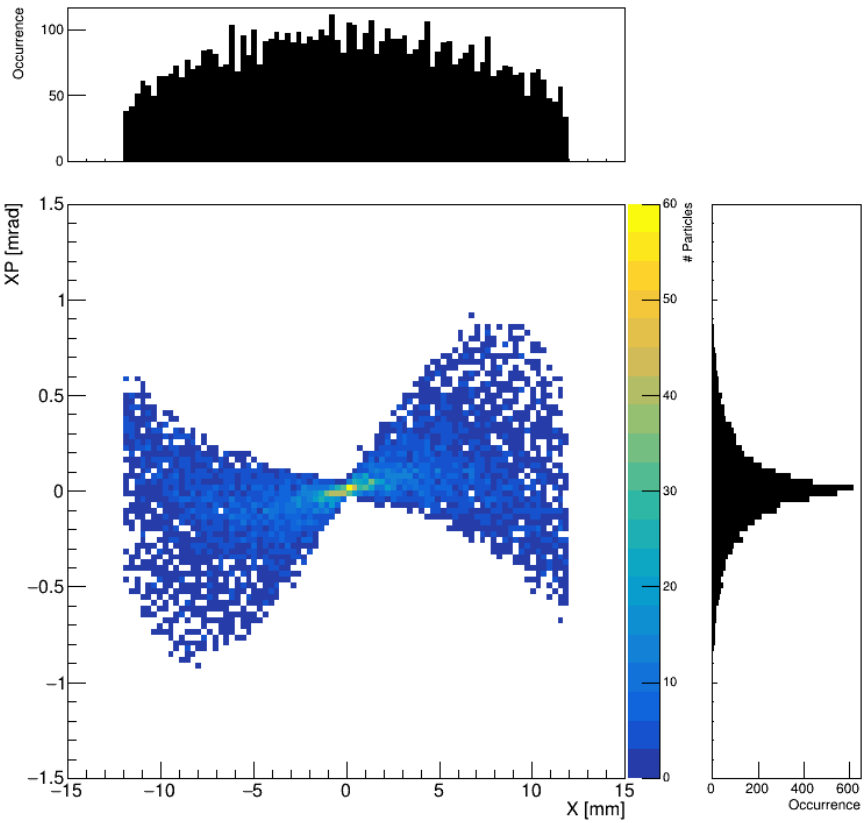
Sampled Beam: Collimator 2 End



β_x [m]	32.91 ± 0.26
α_x	0.28 ± 0.01
ϵ_x [m rad]	$2.41e-06 \pm (3.05e-08)$
β_y [m]	32.64 ± 0.26
α_y	0.27 ± 0.01
ϵ_y [m rad]	$2.45e-06 \pm (3.09e-08)$

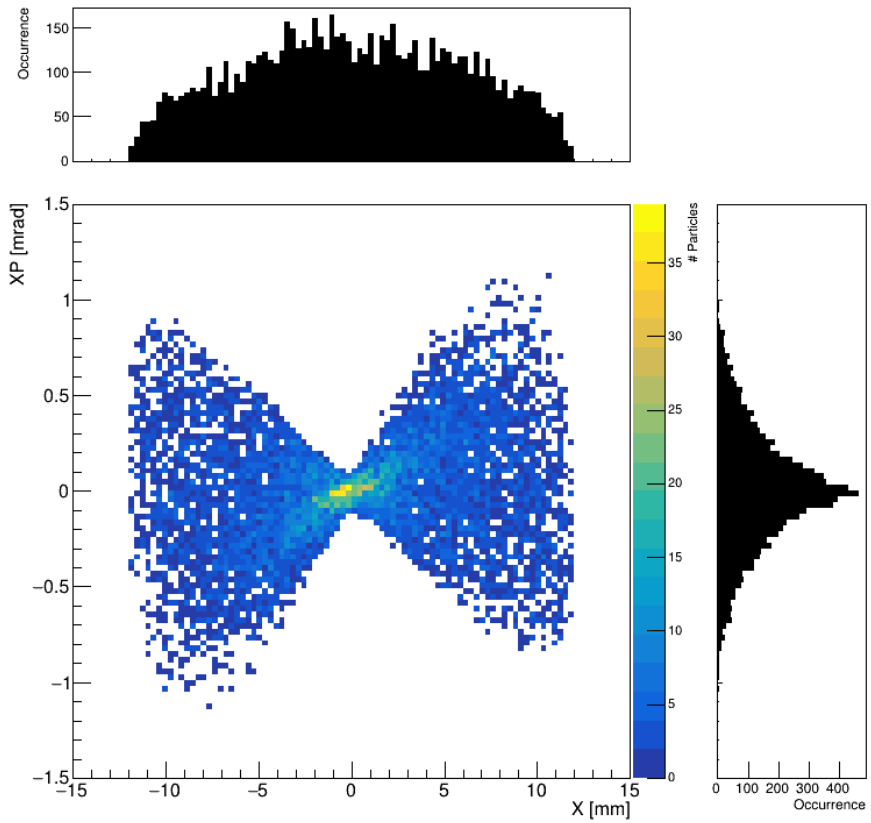
Exit to Collimator 3 (Arc) (S = 12.98 m) – need to verify

Ideal Beam: Collimator 3 End



β_x [m]	26.69 ± 0.27
α_x	-0.38 ± 0.02
ϵ_x [m rad]	$1.48e-06 \pm (2.18e-08)$
β_y [m]	0.03 ± 0.0
α_y	-0.49 ± 0.01
ϵ_y [m rad]	$2.01e-06 \pm (6.91e-08)$

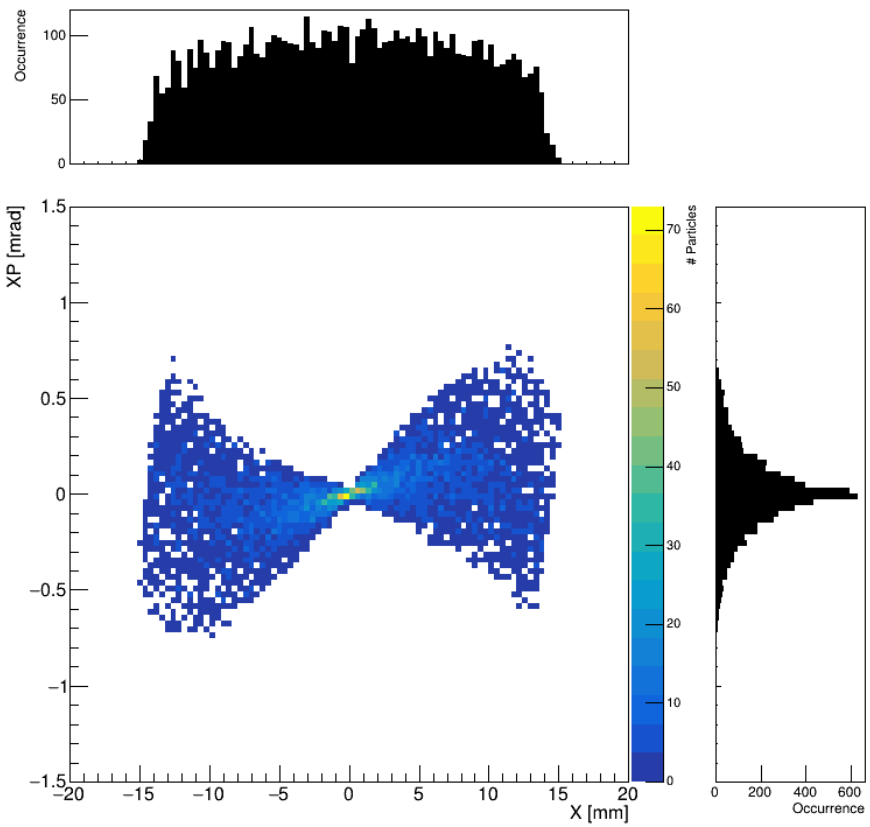
Sampled Beam: Collimator 3 End



β_x [m]	19.02 ± 0.15
α_x	-0.19 ± 0.01
ϵ_x [m rad]	$1.80e-06 \pm (2.24e-08)$
β_y [m]	0.04 ± 0.0
α_y	-0.69 ± 0.01
ϵ_y [m rad]	$2.63e-06 \pm (1.87e-07)$

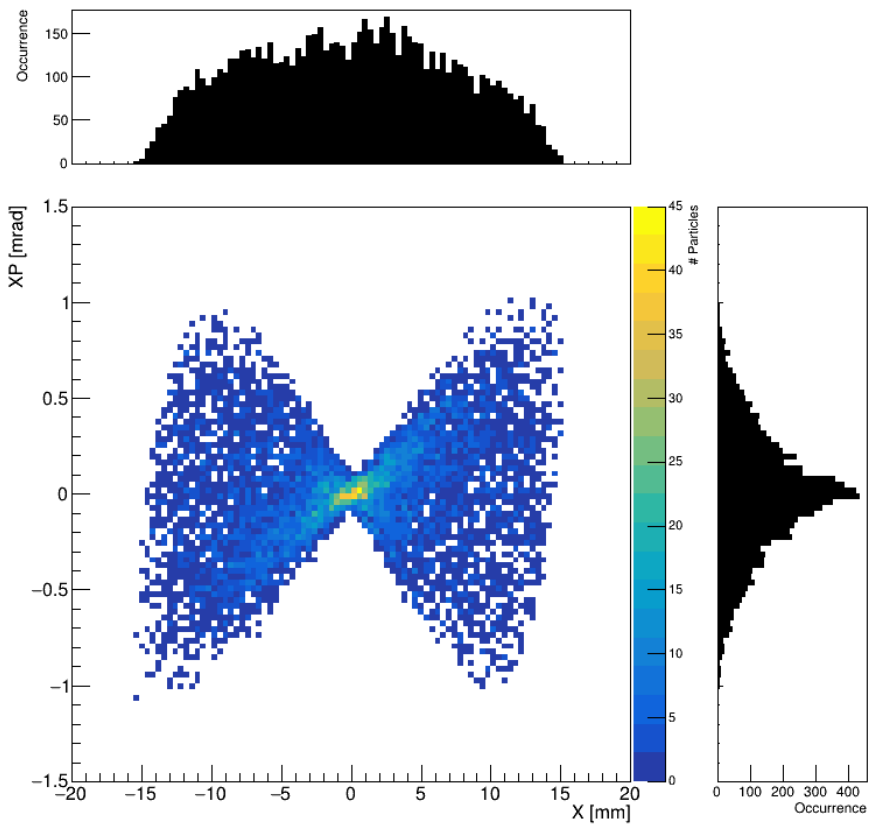
End of Stage 1 (S = 17.275 m) – need to verify

Ideal Beam: Stage 1 End



β_x [m]	38.67 ± 0.38
α_x	-0.36 ± 0.02
ϵ_x [m rad]	$1.57e-06 \pm (2.29e-08)$
β_y [m]	41.54 ± 0.46
α_y	0.21 ± 0.02
ϵ_y [m rad]	$2.05e-06 \pm (3.49e-08)$

Sampled Beam: Stage 1 End



β_x [m]	22.74 ± 0.19
α_x	-0.19 ± 0.01
ϵ_x [m rad]	$2.29e-06 \pm (2.78e-08)$
β_y [m]	28.84 ± 0.26
α_y	0.37 ± 0.01
ϵ_y [m rad]	$3.00e-06 \pm (3.92e-08)$