Centre for the Clinical
Application of Particles

Nuclear diagnostics and Magnetic Resonance Imaging

Lecture 1: introduction, nuclear diagnostics (I)

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`Drawing heavily on material prepaared in 2018/19 by J. Pozimski`

Outline

Section 1

Introduction

Aims

With the course I aim to:

- Provide students with a general overview of the physical principles that underlie nuclear diagnostics (ND) and magnetic resonance imaging (MRI);
- Allow students to appreciate the factors that influence the development of contrast in ND and MRI; and
- Give students the means to estimate the resolution, speed, and sensitivity of ND and MRI imaging modalities.

Objectives 1: radionuclides and gamma camera

At the end of the course students will be able to:

- Explain the principal methods used for the production of radionuclides;
- Explain and apply the definitions of activity, half-life and decay constant;
- Calculate the energies of the particles involved in radioactive decay in a given situation;
- Explain the principal radioactive decay pathways and the application of the radiation produced to medical imaging;
- Explain how a gamma camera operates and discuss the main parameters that affect its performance;
- Calculate the resolution and efficiency of a gamma camera in a particular situation; and
- Explain the different types of event that can be recorded by using a gamma camera and discuss how these affect the image.

Objectives 2: SPECT and PET

At the end of the course students will be able to:

- Explain how Single Photon Emission Computed Tomography (SPECT) is performed;
- Explain the methods typically used in the reconstruction of a SPECT images;
- Explain how a Positron Emission Tomography (PET) image is produced and discuss the principal limitations of PET;
- Discuss the different types of detection events recorded by a PET system and how they affect the image;
- Calculate the resolution, detection efficiency, detection rate, and coincidence rate for a particular PET scanner;
- Explain how to compensate for attenuation and unwanted random and scatter events in PET; and
- Describe and discuss the main differences between 2D and 3D PET image acquisition

Objectives 3: Magnetic resonance imaging

At the end of the course students will be able to:

- Describe the principles of nuclear magnetic resonance (NMR);
- Discuss how relaxation mechanisms contribute to the generation of contrast in magnetic resonance imaging (MRI);
- Describe techniques for spatial localisation of the NMR signals in MRI;
- Distinguish between phase and frequency encoding is used in MRI to enable the spatial localisation of the NMR signal;
- Use the Fourier-transform technique to exploit ‘k-space’ to describe MRI images in this space;
- Connect the receiver bandwidth, acquisition time, and sampling frequency to the field of view and resolution;
- Identify the causes of the most common image artefacts;
- Show how the signal-to-noise ratio (SNR) is affected by the choice of data acquisition parameters;
- Recall the major hardware components necessary for MRI;
- Recall, devise, and interpret typical MRI pulse sequences and discuss the changes in image contrast induced by changes in the pulse sequences; and
- Calculate the optimal ‘TR’ and ‘TE’ to maximise image contrast for ‘proton density’, ‘T1’, and ‘T2’ weighted imaging.

Recommended books

Physics in Nuclear Medicine
S. Cherry, J.A. Sorenson, M.E. Phelps
Central Library: 616.075 CHE

The Essential Physics of Medical Imaging
J. Bushberg, J. Seibert, E. Leidholdt, J. Boone
Central Library: 616.075 BUS

MRI: the basics
R. Hasgemi, W. Bradley, C. Lisanti
Central Library: 616.075 HAS

Section 2

Nuclear diagnostics (I)

What is ‘nuclear diagnostics’?

Imaging using radio-isotopes:

- Introduce radionuclide into the body:
  - A ‘radiotracer’ is usually introduced into the body on a ‘radiophamaceutical’;
  - Detectors external to body used to ‘image’ distribution of radiotracer within body;
  - Spatial distribution depends on ‘take-up’ of radiopharmaceutical in tissue.
- ND usually ‘emission imaging’ . . .
  to be contrasted with conventional x-ray imaging which is ‘transmission imaging’.

Sensitivity!

- Goal is to use such a small amount of radiotracer that biological system under investigation is not perturbed.

Radiopharmaceuticals

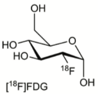
Enormous field! Beyond the scope of this course.

Consider the radiopharmaceutical as a means of delivering an isotope to a particular location:

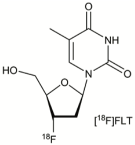
- Compound tagged with a radionuclide;
- Accumulation, rate of uptake and clearance related to physiological, biochemical and/or molecular processes;
- Carrier molecule designed to target organ or function;
- Radionuclide required to produce emissions that can be detected outside the body;:
  - Require penetrating radiation: primary gammas, photons produced in positron annihilation;
- Radiouclides delivered in tiny amounts: nanograms:
  - Require sensitive detectors.
- Administration:
  - Intravenous, inhalation, subcutaneous, oral

Radiopharmaceuticals: an example

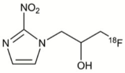
Delivering ^{18}F, and exploitation in diagnosis of Alzheimer's disease using PET.



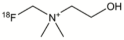
[^{18}F]FDG



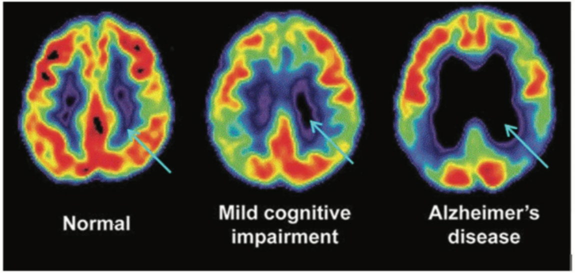
[^{18}F]FLT



[^{18}F]FMISO



[^{18}F]FCH

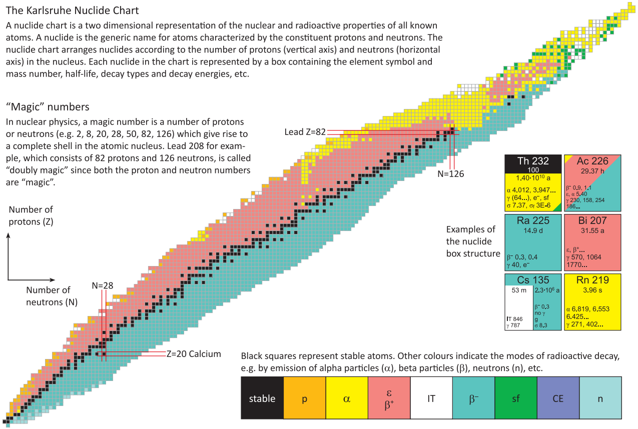


http://real.mtak.hu/41305/1/225_247_BRC_2015_Vol_2.pdf

Radioactivity ‘101’

- Radioactivity is an intrinsic property of unstable nuclei;
- Quantum-mechanical process: uniform probability of decay;
- Radioactive decays:
    - Electromagnetic: de-excitation yielding photons . . . gammas;
    - Weak:
        - $N \to N' + e^- + \bar{\nu}_e$
        - $N \to N' + e^+ + \nu_e$ ← the basis of PET.
    - Strong: e.g. α-decay; not of use for ND.
- Isotopes have the same number of protons;
- Isotones have the same number of neutrons;
- Isobars have the same number of nucleons.

Nuclei and decay modes



The Karlsruhe Nuclide Chart

A nuclide chart is a two dimensional representation of the nuclear and radioactive properties of all known atoms. A nuclide is the generic name for atoms characterized by the constituent protons and neutrons. The nuclide chart arranges nuclides according to the number of protons (vertical axis) and neutrons (horizontal axis) in the nucleus. Each nuclide in the chart is represented by a box containing the element symbol and mass number, half-life, decay types and decay energies, etc.

"Magic" numbers

In nuclear physics, a magic number is a number of protons or neutrons (e.g. 2, 8, 20, 28, 50, 82, 126) which give rise to a complete shell in the atomic nucleus. Lead 208 for example, which consists of 82 protons and 126 neutrons, is called "doubly magic" since both the proton and neutron numbers are "magic".

Black squares represent stable atoms. Other colours indicate the modes of radioactive decay, e.g. by emission of alpha particles (α), beta particles (β), neutrons (n), etc.

https://www.epj-n.org/articles/epjn/full_html/2019/01/epjn180014/epjn180014.html

Radioactive decay

For a single nucleus, probability of decay per unit time is constant:

$$\frac{dP}{dt} = -\lambda$$

So, probability that nucleus will eventually decay is 1.
If there are N nuclei at time t:

$$\tfrac{dN}{dt} = -\lambda N \qquad \ldots \text{and so} \ldots \qquad N(t) = N(0)\exp(-\lambda t)$$

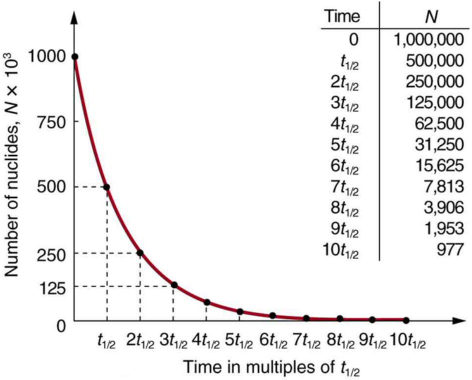
λ is the decay constant. The 'halflife', $t_{\frac{1}{2}}$, is related to the 'lifetime', τ, and λ by:

$$t_{\frac{1}{2}} = \frac{\ln(2)}{2}\tau = \frac{\ln(2)}{2}\frac{1}{\lambda}$$

Activity

Activity: $A = \left|\frac{dN}{dt}\right| = \gamma N$

- SI unity of activity: Becquerel (Bq):
    - $1\,\mathrm{Bq} = 1$ decay per second
- Curie (Ci):
    - $1\,\mathrm{Ci} = 3.7 \times 10^{10}\,\mathrm{Bq}$



Time	N
0	1,000,000
$t_{1/2}$	500,000
$2t_{1/2}$	250,000
$3t_{1/2}$	125,000
$4t_{1/2}$	62,500
$5t_{1/2}$	31,250
$6t_{1/2}$	15,625
$7t_{1/2}$	7,813
$8t_{1/2}$	3,906
$9t_{1/2}$	1,953
$10t_{1/2}$	977

`https://courses.lumenlearning.com/physics/chapter/31-5-half-life-and-activity/`

β decay

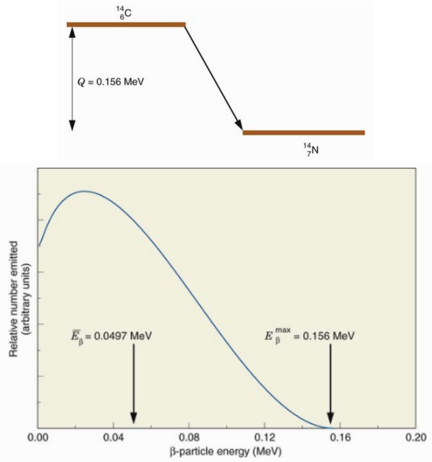
Underlying process: $n \rightarrow p + e^- + \bar{\nu}_e$

$$ {}^A_Z X \xrightarrow{e^-(\beta^-)} {}_{Z+1}{}^A Y $$

$$ {}^A_Z X + e^- \rightarrow {}_{Z+1}{}^A Y + e^- + \bar{\nu}_e $$

Kinematics requires neutral atoms, hence e^- on LHS.

Continuous electron-energy spectrum due to 3-body decay.



(β, γ) decay

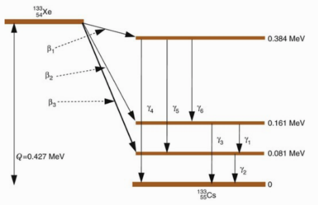
β absorbed in short distance; no use for ND.
Exploit de-excitation γs.

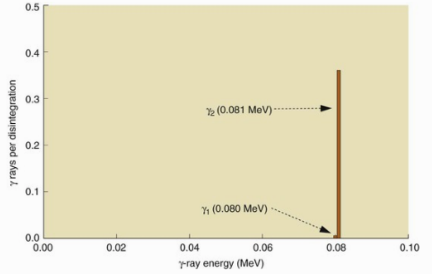
$${}^{A}_{Z}X \xrightarrow{\beta^-} {}^{A}_{Z+1}Y^* \xrightarrow{\gamma} {}^{A}_{Z+1}Y$$

β-decay leaves Y in excited state—indicated by ‘$*$’.

Discrete photon energy or energies.

Electron from β decay absorbed in tissue.





Isomeric transition (IT)

Daughter nuclide produced in long-lived, ‘metastable’ state.
Again, exploit de-excitation γs.

$${}^{A}_{Z}X \xrightarrow{\beta^-} {}^{Am}_{Z+1}Y \xrightarrow{\gamma} {}^{A}_{Z+1}Y$$

β-decay leaves Y in metastable excited state—indicated by ‘m’.

Discrete photon energy or energies.

IT identical to decay by γ production, except that lifetime is much longer.

Important example:

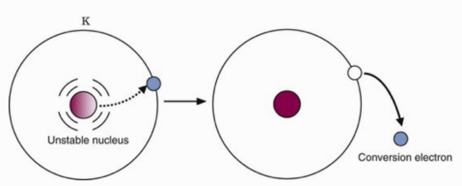
$$\boxed{\begin{array}{ccc} {}^{99}Mo \xrightarrow{\beta^-} {}^{99m}Tc & & \xrightarrow{\gamma} {}^{99}Tc \\ \tau_{1/2} = 66\text{ hrs} & & \tau_{1/2} = 6\text{ hrs} \end{array}}$$

Halflife of ^{99}Tc is 211 millenia:

- Decay rate of ^{99}Tc is low; but
- Clearly important to ensure ‘waste’ radionuclide is excreted.

Internal conversation (IC)

$$ {}^{A}_{Z}X^{*} \xrightarrow{\text{IC}} {}^{A}_{Z}X + e $$



Nucleus in an excited state or a meta-stable excited state may decay such that:

- Energy from nuclear is transition transfered to orbital electron which is emitted;
- Electron emitted has an energy typical of a nuclear transition;
- Orbital vacancy is filled creating characteristic X-rays or Auger electrons.

In a sense, the γ that might have been emitted is ‘internally converted’ into an electron.

Positron emission

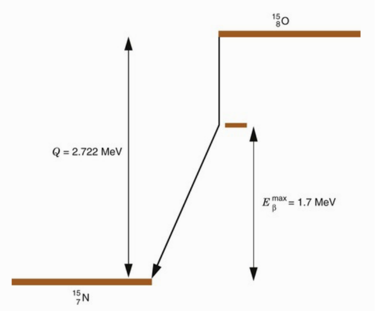
Underlying process: $p \rightarrow n + e^{+} + \nu_e$

$$ {}^{A}_{Z}X \xrightarrow{e^{+}(\beta^{+})} {}_{Z-1}^{A}Y $$

$$ {}^{A}_{Z}X \rightarrow {}_{Z-1}^{A}Y + e^{-} + e^{+} + \nu_e $$

Kinematics requires neutral atoms,
hence e^{-} on RHS.
[Orbital electron ‘lost’ from atom after decay]

Continuous electron-energy spectrum due to 3-body decay.



$$
\begin{aligned}
Q &= 2.72\,\mathrm{MeV} \\
&= 2m_e c^2 + E_{\beta}^{\mathrm{max}} \\
&= 1.022\,\mathrm{MeV} + 1.7\,\mathrm{MeV}
\end{aligned}
$$

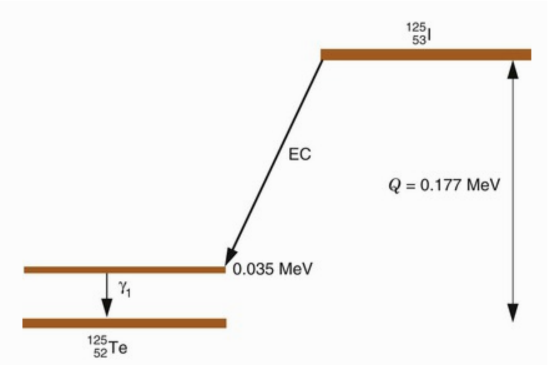
Electron capture (EC) and (EC, γ)

Underlying process: $p + e^- \rightarrow n + \nu_e$
Orbital electron is captured by nucleus; transforms proton to neutron.

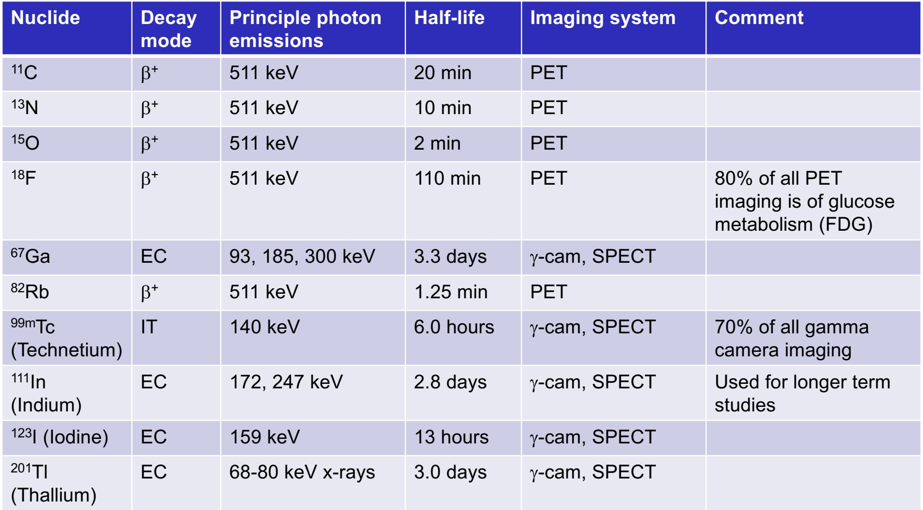
$${}^A_Z X \xrightarrow{\text{EC}} {}_{Z-1}^{A} Y$$

Orbital vacancy leads to the emission of characteristic X-rays.
Only useful for imaging if X-rays are of high enough energy.

γs or conversion electrons may be emitted:
γs have 'MeV-scale' energies are are more likely to be of use for ND.



Commonly used radionuclides for imaging in ND



Nuclide	Decay mode	Principle photon emissions	Half-life	Imaging system	Comment
^{11}C	β^+	511 keV	20 min	PET	
^{13}N	β^+	511 keV	10 min	PET	
^{15}O	β^+	511 keV	2 min	PET	
^{18}F	β^+	511 keV	110 min	PET	80% of all PET imaging is of glucose metabolism (FDG)
^{67}Ga	EC	93, 185, 300 keV	3.3 days	γ-cam, SPECT	
^{82}Rb	β^+	511 keV	1.25 min	PET	
^{99m}Tc (Technetium)	IT	140 keV	6.0 hours	γ-cam, SPECT	70% of all gamma camera imaging
^{111}In (Indium)	EC	172, 247 keV	2.8 days	γ-cam, SPECT	Used for longer term studies
^{123}I (Iodine)	EC	159 keV	13 hours	γ-cam, SPECT	
^{201}Tl (Thallium)	EC	68-80 keV x-rays	3.0 days	γ-cam, SPECT	