Nuclear diagnostics and Magnetic Resonance Imaging

Lecture 7: Magnetic Resonance Imaging: introduction

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Outline

Section 1

Magnetic resonance imaging: introduction and principles

'Guilt-free' imaging

Whole-body imager, Star Trek style

Nuclear diagnostics and X-ray imaging:

- Image constructed using ionising radiation
- Necessarily delivers dose to patient
- Dose implies risk of initiating disease

Magnetic resonance imaging (MRI):

- Image generated by exploiting magnetic moment of H nuclei
- Patient immersed in magnetic field
- No permanent harmful effects reported

Nuclear magnetic moment

Proton (and neutron) magnetic moment:

- Nucleons each have spin of $\frac{1}{2}$
- Magnetic moment generated by nuclear charge

  Contributions to nuclear spin arise from quarks and gluons. Quantitative explanation of nuclear magnetic moment is an active area of research
- For NMR and MRI critical point is that the magnetic moment, $\boldsymbol{\mu}$, is related to the nuclear spin, $\mathbf{s}$ by:

$$\boldsymbol{\mu} = \gamma \mathbf{s}$$

where γ is the "gyromagnetic" ratio

Nuclear magnetic resonance

Effect of uniform magnetic field **B**:

- **B** provides "quantisation axis":
  $\Rightarrow$ nuclear dipoles align with magnetic field
- For proton spin is $\frac{1}{2}$, so only two states:
  Spin "up" and spin "down"
- Energy splitting; 2 energy levels:
  - Lower energy level has magnetic moment parallel to magnetic field
  - Higher energy level has magnetic moment anti-parallel to magnetic field
- Resonance:
  - Call energy splitting ΔE
  - Transitions between the two energy levels cause absorption or emission of electromagnetic (em) radiation for which $\Delta E = h\nu$
  - Resonance occurs when em radiation of frequency ν is injected

Magnetic resonance imaging

Magnetic resonance imaging (MRI) exploits this resonance

Steps:

- Apply uniform magnetic field, align proton (^{1}H) spins
- Apply radiation, at exactly ν, to cause transitions between "spin up" & "spin down" states
- Turn off the radiation . . . and . . .
- "Listen" for radiation at exactly ν as the spins realign

Brilliant! Simple principle and elegant technique. Now exploited in exquisitely sophisticated imaging systems.

The physical principles

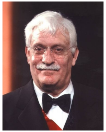
1938: I. Rabi: Discovered nuclear magnetic resonance
Nobel Prize 1944

1946: F. Bloch & E. Purcell: Developed methods that allow precision methods using NMR
Nobel Prize 1952

1955/56: E. Odeblad & G. Lindström: Applied NMR to living cells from animal tissue

1968: J.A. Jackson and W.H. Langham: First NMR measurements from living animals

Cancerous and normal cells differ



Raymond Damadian

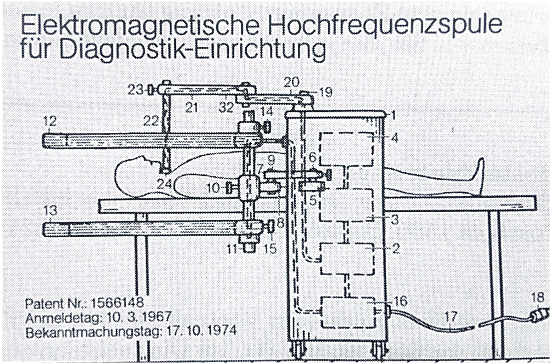
Relaxation times that characterise recovery of ground-state magnetisation shown to differ between normal and tumour cells

Tumor Detection by Nuclear Magnetic Resonance
Author(s): Raymond Damadian
Source: *Science*, New Series, Vol. 171, No. 3976 (Mar. 19, 1971), pp. 1151-1153
Published by: American Association for the Advancement of Science
Stable URL: https://www.jstor.org/stable/1730608
Accessed: 01-03-2020 09:22 UTC

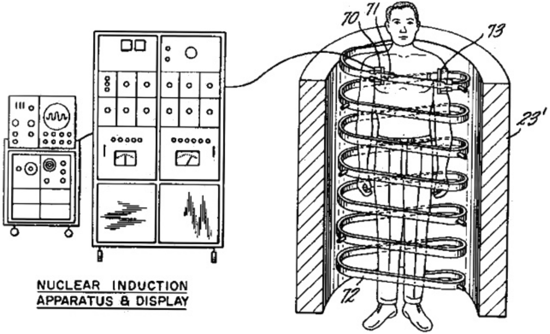
REFERENCES
Linked references are available on JSTOR for this article:
https://www.jstor.org/stable/1730608?seq=1&cid=pdf-reference#references_tab_contents
You may need to log in to JSTOR to access the linked references.

Early proposals for MRI scanners

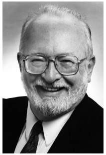
Alexander Ganssen; patent 1967



Raymond Damadian; patent 1972



Spatial localisation using magnetic-field gradients



Paul Lauterbur

Superimpose field gradient on main uniform magnetic field. Incident em radiation at frequency ν only resident in a particular location in subject

Nature Vol. 242 16 March 1973

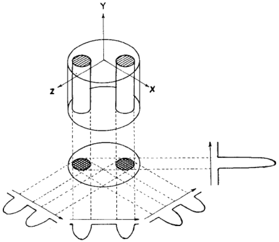


Fig. 1 Relationship between a three-dimensional object, its two-dimensional projection along the Y-axis, and four one-dimensional projections at 45° intervals in the XZ-plane. The arrows indicate the gradient directions.

Fig. 2 Proton nuclear magnetic resonance zeugmatogram of the object described in the text, using four relative orientations of object and gradients as diagrammed in Fig. 1.

Rapid, "snap-shot" MRI

Use of "echo planar imaging" to allow fast "snap-shot" imaging required active screening of fields created by currents induced in cryostat walls

Peter Mansfield

P. Mansfield, Nobel Lecture 2003

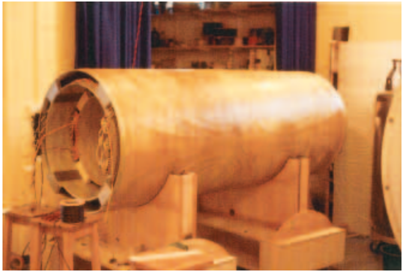


Figure 2. Photograph of a doubly screened active magnetic shielded gradient coil set for insertion in the super-conductive magnet of Figure 1.

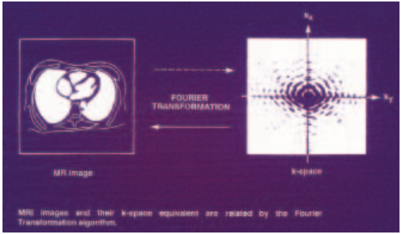


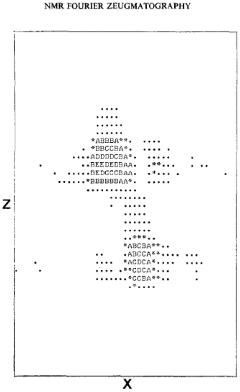
Figure 3. Diagram of a slice through the mediastinum showing the two lung fields and heart mass, also shown is the Fourier transform of this real-space image to the k-space map. (Reproduced with permission from M K Stehling, R Turner and P Mansfield, SCIENCE 253, 43–50 (1991).)

NMR zeugmatography

1975: A. Kumar, D. Welti, R. Ernst

Application of Fourier techniques to the reconstruction of images

Journal of Magnetic Resonance, Vol 18, P 69–83(1975)



zeug·ma·tog·ra·phy (zūg'mă-tog'ră-fē),
Term coined by Lauterbur in 1972 for the joining of a magnetic field and spatially defined radiofrequency field gradients to generate a two-dimensional display of proton density and relaxation times in tissues, the first nuclear magnetic resonance image.

State of the art

Theoretical description; a hybrid of quantum and classical

Nuclear magnetic resonance & MRI are both inherently quantum mechanical effects:

- Signal is generated by manipulating the *spins* of hydrogen nuclei:
  - Spin is postulated to explain hyperfine structure, Stern-Gerlach experiment, ...
  - Understood theoretically through the symmetries of space and time
- Magnetic moment of proton, $\boldsymbol{\mu}$, is related to the proton spin, **s**, by:

$$\boldsymbol{\mu} = \gamma \mathbf{s}$$

  where γ is the "gyromagnetic ratio"

Hybrid, quantum/classical treatment:

- Quantum mechanics: energy splitting and population in ground and excited state
- Classical: magnetisation vector, its precession, and the manipulation of the magnetisation vector to generate the signals used for imaging

Interaction of nuclear magnetic dipole with uniform magnetic field

The contribution, $\delta\mathcal{U}$, to the potential energy of a proton immersed in a magnetic field, **B**, is given by:

$$\delta\mathcal{U} = -\mathbf{B} \cdot \boldsymbol{\mu}$$

Lets consider a proton which, in the absence of a magnetic field has energy E. Applying the magnetic field introduces $\delta\mathcal{U}$ into the Schrödinger equation resulting in a splitting of the proton energy level such that $E \rightarrow E'$ given by:

$$E' = E \pm E_{m_s}$$

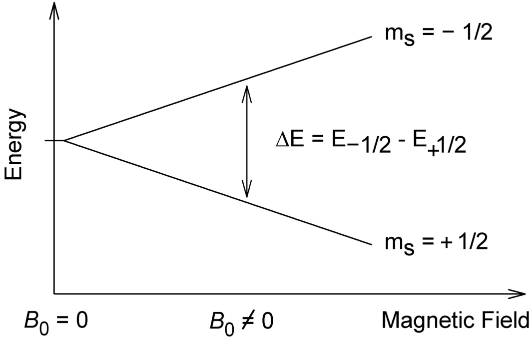
where

$$E_{m_s} = -m\gamma\hbar B_0$$

where m is the quantum number associated with the component of the proton spin parallel to **B**, $\hbar$ is Planck's constant divided by 2π, and B_0 is the magnitude of **B**
For the proton:

$$m_s = \pm\frac{1}{2}$$

Larmor equation



ΔE, splitting between two levels with $m_s = \pm\frac{1}{2}$:

$$\Delta E = \gamma \hbar B_0$$

Planck's law relates energy splitting to the angular frequency, ω, of the radiation required to excite the transition, therefore:

$$\Delta E = \hbar \omega$$

Writing ω in terms of γ and B_0 yields the Larmor equation:

$$\omega = \gamma B_0$$

Gyromagnetic ratios of some nuclei

Definition of gyromagnetic ration, γ:

> *The gyromagnetic ratio, γ, of a particle or system is the ratio of its magnetic dipole moment to its angular momentum*

For charged body of charge q, mass m rotating about an axis of symmetry:

$$\gamma = \frac{qe}{2m}$$

where e is the magnitude of the charge on the electron

For proton, $q = 1$, $m = m_p$, the proton mass.

$\bar{\gamma}$ is sometimes used instead of γ:

$$\bar{\gamma} = \frac{\gamma}{2\pi}$$

nucleus	γ (rad MHz T^{-1})	$\bar{\gamma} = \gamma / 2\pi$
^{1}H	267.513	42.576
^{2}H	41.065	6.536
^{3}He	203.789	32.434
^{7}Li	103.962	16.546
^{13}C	67.262	10.705
^{14}N	19.331	3.077
^{15}N	27.116	-4.316
^{17}O	36.264	5.772
^{19}F	251.662	40.053
^{23}Na	70.761	11.262
^{27}Al	69.763	11.103
^{31}P	108.291	17.235
^{57}Fe	8.681	1.382
^{63}Cu	71.118	11.319
^{67}Zn	16.767	2.669
^{129}Xe	73.997	11.777

Examples

Larmor equation: $\omega = \gamma B_0 \quad \Rightarrow \quad \nu = \bar{\gamma} B_0$

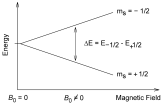
For hydrogen nucleus, ^{1}H, $\bar{\gamma} = 42.58\,\mathrm{MHz/T}$

What is the resonance frequency for ^{1}H when:

- $B_0 = 1.5\,\mathrm{T}$?
- $B_0 = 3.0\,\mathrm{T}$?

What are the corresponding values for the energy splittings $\Delta E = h\nu$, where h is Planck's constant?

Populations in the two spin states



^{1}H in tissue in thermal equilibrium, so, partition between the populations in the two spin states follows the Boltzmann distribution:

$$\frac{N_+}{N_-} = \exp\left(-\frac{\Delta E}{k_{\rm B} T}\right)$$

where N_+ and N_- are the number of ^{1}H in $+\Delta E$ and $-\Delta E$ states respectively, $k_{\rm B}$ is Boltzmann's constant, and T is the temperature
For the human body, $k_{\rm B} T \approx 25.7\,$meV, so:

$$\Delta E << k_{\rm B} T$$

Therefore, expanding the exponential and rearranging:

$$N_- - N_+ \approx N_S \frac{\Delta E}{2 k_{\rm B} T}$$

Magnetisation

Substituting for ΔE

$$N_- - N_+ \approx N_S \frac{\Delta E}{2 k_{\mathrm{B}} T} = N_S \frac{\gamma h B_0}{4\pi k_{\mathrm{B}} T}$$

For $B_0 = 1.5\,\mathrm{T}$:

$$\frac{N_- - N_+}{N_S} \approx \frac{42.58 \times 10^6 \times 6.6 \times 10^{-34} \times 1.5}{2 \times 1.38 \times 10^{-23} \times 300}$$

$$\approx 4.5 \times 10^{-6}$$

i.e. only 4.5 in a million protons in the body are available for activation in MRI at $B_0 = 1.5\,\mathrm{T}$

Bulk magnetisation is measurable

Population-density "mismatch" of $\approx 3\,$ppm per Tesla arises due to fact that energy splitting is small compared to $k_{\mathrm{B}}T$

Bulk magnetisation still measurable because 1 gram of water contains 10^{22} ^{1}H

Section 2

Lecture summary

Summary

MRI technique is based on manipulation of ^{1}H spins; a quantum-mechanical effect

MRI can be described using a hybrid quantum-mechanical/classical treatment

Application of magnetic field B_0 causes splitting ΔE between the two spin states of an ^{1}H nucleus:

$$\Delta E = \hbar\omega$$

where ω is the Larmor frequency:

$$\omega = \gamma B_0$$

Population of lower energy state of ^{1}H is $\approx 3\,$ppm per Tesla greater than higher energy state