Nuclear diagnostics and Magnetic Resonance Imaging

Lecture 8: Magnetic Resonance Imaging: manipulation of magnetisation

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Outline

1. **Manipulation of magnetisation**
   - Classical derivation of the Larmor equation
   - Rotating $\mathbf{M}$; the RF magnetic field $\mathbf{B}_1$
   - Free induction decay

2. **Lecture summary**

Section 1

Manipulation of magnetisation

The Larmor equation and bulk magnetisation; reprise

The quantum mechanical treatment presented in lecture 7 led to the **Larmor equation**:

$$\omega = \gamma B_0$$

ω is the Larmor frequency, B_0 the magnitude of the magnetic field, γ the gyromagnetic ratio

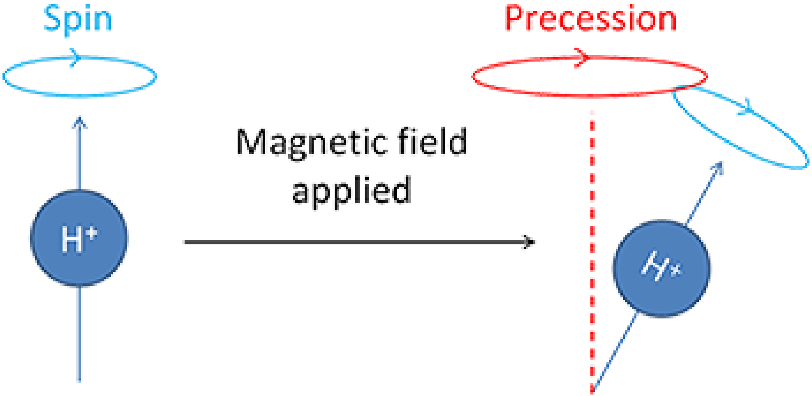
ω is the resonant frequency in an external magnetic field

The **bulk magnetisation** was obtained by considering the partition between the two energy states of the ^{1}H nuclei in the magnetic field:

$$N_- - N_+ \approx N_S \frac{\Delta E}{2 k_\mathrm{B} T} = N_S \frac{\gamma \hbar B_0}{2 k_\mathrm{B} T}$$

where the notation is that defined in lecture 7

Classical magnetic moment in magnetic field

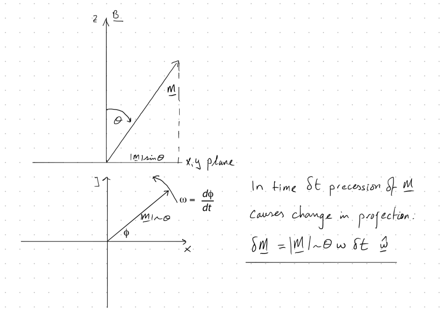


Magnetic moment that makes an angle with a magnetic field will precess around the magnetic-field axis.

Classical derivation of the Larmor equation

Classically, a magnetic moment, **M**, in a magnetic field **B**, experiences a torque given by the Bloch equation:

$$\frac{d\mathbf{M}}{dt} = \gamma\left(\mathbf{M} \times \mathbf{B}\right)$$



M makes an angle θ w.r.t. **B**. So:

$$\mathbf{M} \times \mathbf{B} = MB_0 \sin\theta\hat{\boldsymbol{\omega}}$$

So:

$$\frac{d\mathbf{M}}{dt} = \gamma MB_0 \sin\theta\hat{\boldsymbol{\omega}} = M\sin\theta\omega\hat{\boldsymbol{\omega}}$$

Which gives the Larmor equation:

$$\omega = \gamma B_0$$

Examples—from lecture 7

Larmor equation: $\omega = \gamma B_0 \quad \Rightarrow \quad \nu = \gamma\!\!\!\!- B_0$

Energy splitting: $\Delta E = \hbar\omega \quad \Rightarrow \quad \Delta E = h\nu$

$h = 4.1357 \times 10^{-15}\,\mathrm{eV\,s}$

For hydrogen nucleus, ^{1}H, $\gamma\!\!\!\!- = 42.58\,\mathrm{MHz/T}$

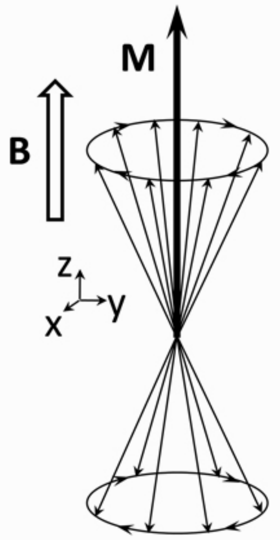
Calculating the values of ν and ΔE yields:

Magnetic field B_0 (T)	Larmor frequency (MHz)	ΔE (eV)
1.5	63.87	2.64E-07
3.0	127.74	5.28E-07

For comparison:

- FM radio waveband runs from 88.1 MHz to 108.1 MHz;
- $k_{\mathrm{B}} T = 2.59 \times 10^{-2}\,\mathrm{eV}$

Larmor precision



Ensemble of ^{1}H nuclei, the majority (by $\approx 3\,\mathrm{ppm\,T^{-1}}$) orientated parallel to **B** precess at equilibrium around **B** at the Larmor angular frequency ω

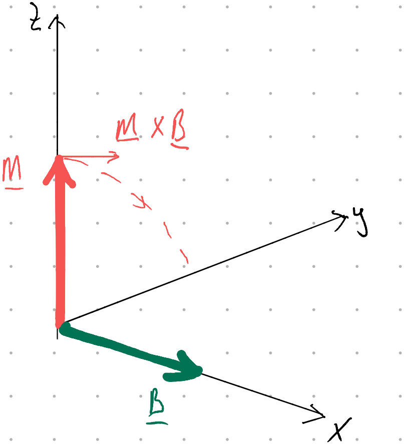
Net magnetisation, **M**, produced is parallel to **B**.

There is no net magnetisation in the transverse (x, y) plane; sum of all contributions cancel

Result is that there is no change in the magnitude or direction of the magnetisation vector so no RF signal is produced

Key feature of MRI: manipulate **M** so as to produce a measurable RF signal

First, a static example



Consider magnetisation **M** parallel to z axis and **B** parallel to the x axis, as shown

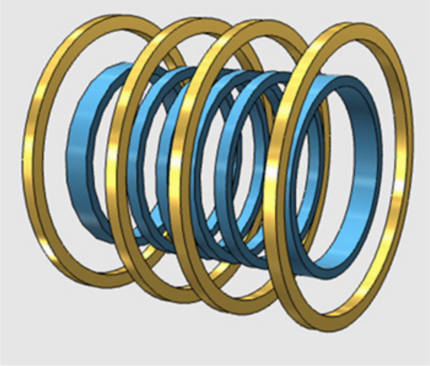
Torque, $\mathbf{M} \times \mathbf{B}$, is therefore parallel to the y axis

Net result is that **M** will precess around the x axis towards the y axis

This is what is done in MRI . . .

Rotating the magnetisation vector in MRI; principle

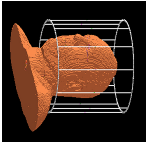
Main field, $\mathbf{B}_0$, produced with solenoid



Induces magnetisation $\mathbf{M}$ parallel to $\mathbf{B}_0$

To rotate $\mathbf{M}$ away from $\mathbf{B}_0$ require magnetic field in transverse (x, y) plane

Call the field in the x, y plane $\mathbf{B}_1$; can be produced with a variety of coil arrangements, e.g. dipole or, more efficient, a "bird cage"



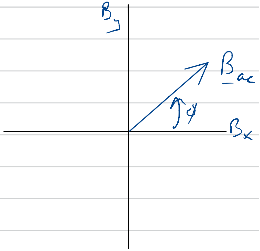
To cause $\mathbf{M}$ to precess require that $\mathbf{M}$ oscillates at the Larmor frequency, ω. I.e. require RF magnetic field $\mathbf{B}_1$

Rotating the magnetisation vector in MRI; mathematics

Take $\mathbf{B}_1$ to be "plane polarised" in x, y such that $B_{1_x} = B_1 \cos(\omega t + \alpha)$ and $B_{1_y} = B_1 \sin(\omega t + \beta)$; α and β are phases

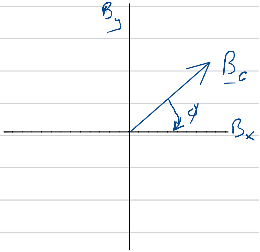
$\mathbf{B}_1$ can be rewritten in terms of two circularly polarised fields:

$\mathbf{B}_{1_{ac}}$; anti-clockwise



$$B_{1_{ac}} = \frac{B_1}{2};\ \phi_{ac} = \omega t + \alpha'$$

$\mathbf{B}_{1_c}$; clockwise



$$B_{1_c} = \frac{B_1}{2};\ \phi_c = \omega t + \beta'$$

Rotating the magnetisation vector in MRI

One of the two counter rotating fields will rotate in the same direction as the nuclear precession

In the frame that is co-rotating with the precession of the net magnetisation vector the magnetic field will appear stationary in the transverse (x, y) plane. Call the co-rotating field B_1^+

B_1^+ is equal to either $B_{1_{ac}}$ or B_{1_c} depending on the direction of $\mathbf{B}_0$

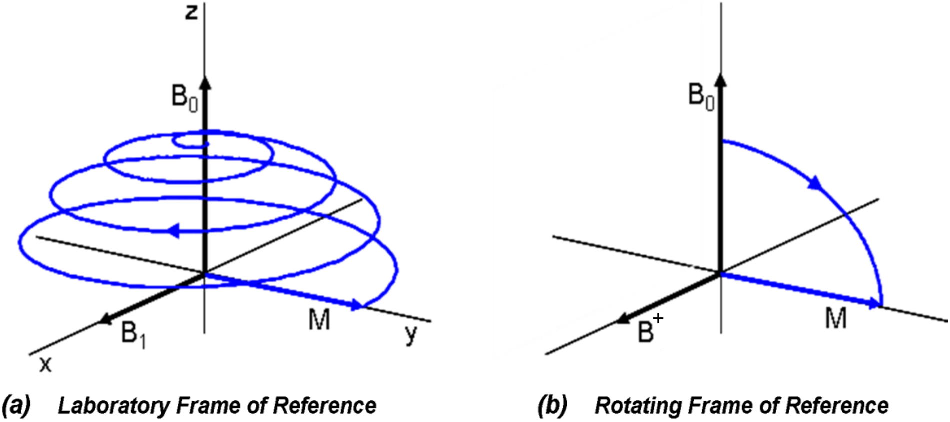
The stationary field will therefore cause $\mathbf{M}$ to precess about a rotating axis in the (x, y) plane

The net result is that $\mathbf{M}$ can be rotated into the x, y plane where it will continue to precess

The precession of $\mathbf{M}$ in the x, y plane gives a detectable RF signal

Rotating the magnetisation vector in MRI

M is initially parallel to $\mathbf{B}_0$



The flip angle

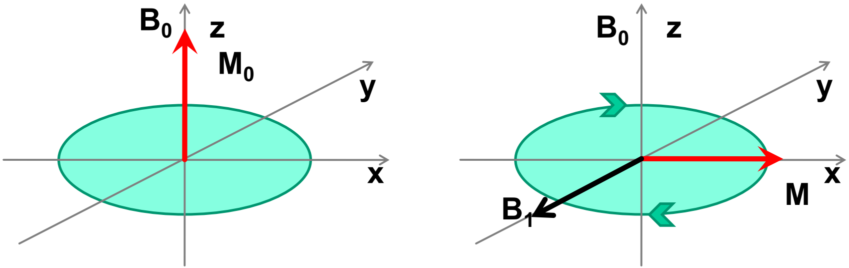
The flip angle, α, is proportional to the magnitude and duration of the RF pulse:

$$\alpha = \gamma B_1 t_P$$

where t_P is the duration of the RF pulse

90° pulse rotates magnetisation into transverse plane where it continues to precess

Effect of 90° RF Pulse



Example: calculating the duration of a 90° pulse

RF transverse magnetic field pulse is applied to rotate **M**

The magnitude of B_1 is $10\,\mu$T (i.e. 10^{-5} T)

At what rate with the **M** rotate away from the $\mathbf{B}_0$ axis?

How long will it take for the flip angle to reach 90°?

Example: calculating the duration of a 90° pulse

Half an answer ... numerical results next time!

The magnitude of B_1 is $10\,\mu$T (i.e. 10^{-5} T)

At what rate with the **M** rotate away from the $\mathbf{B}_0$ axis?

It will rotate at the Larmor frequency, f_1 arising from the field B_1, i.e. $f_1 = \gamma B_1$

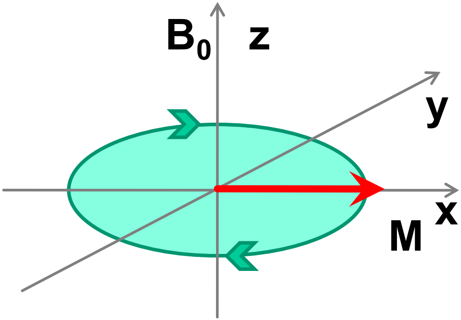
How long will it take for the flip angle to reach 90°?

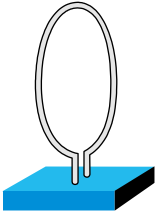
The angle can be obtained by solving the equation: $\frac{\pi}{2} = \gamma B_1 t_P^{90^\circ}$ for $t_P^{90^\circ}$

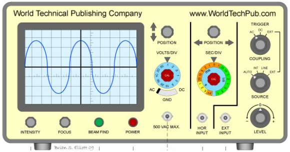
Detection of signal precession of magnetisation vector

M rotated using B_1 RF pulse. If flip angle α is not a multiple of 180°, then, result of B_1 pulse is a component of magnetisation in the x, y plane that is precessing

This yields an RF wave that can be detected

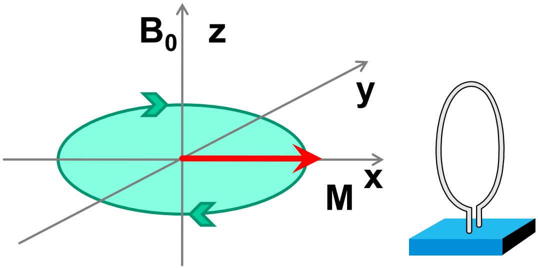


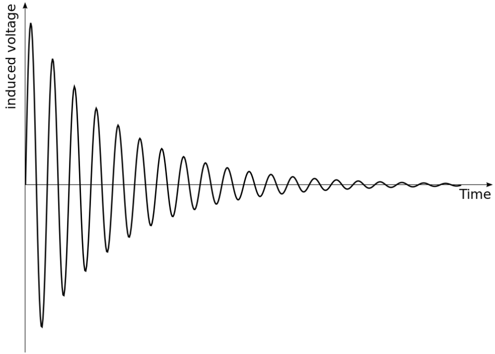




Free induction decay (FID)

Occurs when perturbing field (B_1) is turned off





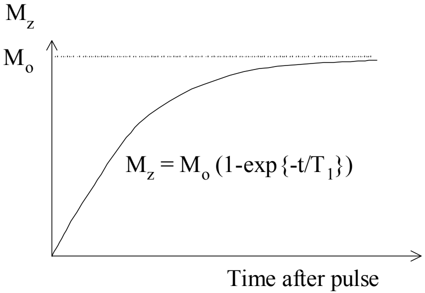
Note; exponential decay of amplitude of transverse magnetisation. Frequency of rotation remains the Larmor frequency corresponding to B_0

Spin-lattice (longitudinal) relaxation

When the B_1 pulse is turned off, the longitudinal magnetisation, M_z, recovers:

$$\frac{dM_z}{dt} = \frac{M_0 - M_z}{T_1} \qquad \Rightarrow M_z(t) = M_0 \left[1 - \exp\left(-\frac{t}{T_1}\right)\right]$$

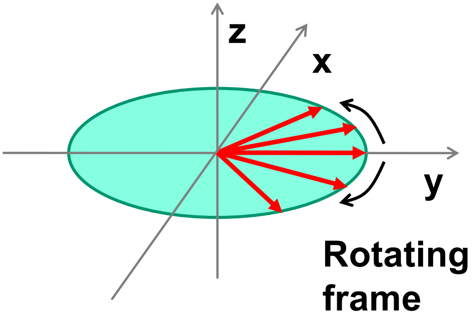
The process is characterised by a time constant T_1



Spin-lattice relaxation:

- ^{1}H spins relax to the low-energy state. Energy released returns to the "lattice" as heat
- Relatively ineffective thermal coupling to ^{1}H nuclei results in T_1 being large, typically $T_1 > 200\,\text{ms}$

Spin-spin (transverse) relaxation



Contributions to M_{xy} smear out (decohere) rapidly

Causes M_{xy} to decay quickly

Some factors that affect the decoherence rate:

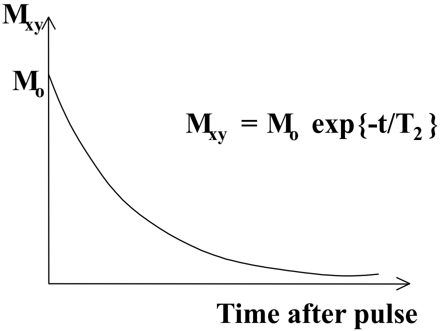
- Resonance frequency changes due to local magnetic fields
- Thermal excitations
- Spin “mobility”
- Presence of large molecules o, paramagnetic ions or molecules, outside interference

Spin-spin (transverse) relaxation

When the B_1 pulse is turned off, transverse magnetisation, decays:

$$\frac{dM_{xy}}{dt} = -\frac{M_{xy}}{T_2} \qquad \Rightarrow M_{xy}(t) = M_0 \exp\left(-\frac{t}{T_2}\right)$$

The process is characterised by a time constant T_2



Spin-spin relaxation:

- ^{1}H spins interact magnetically with their neighbours
- Coupling causes a variety of magnetic fields, causing a variety of precessions
- Effective randomisation of precessional modes leads to efficient depolarisation in transverse plane
- Results in T_2 being comparatively small, typically $T_2 \lesssim 100\,\text{ms}$

Relaxation times for a variety of tissues

Tissue Type	T1 (ms)	T2 (ms)
Adipose tissues	240-250	60-80
Whole blood (deoxygenated)	1350	50
Whole blood (oxygenated)	1350	200
Cerebrospinal fluid (similar to pure water)	4200 - 4500	2100-2300
Gray matter of cerebrum	920	100
White matter of cerebrum	780	90
Liver	490	40
Kidneys	650	60-75
Muscles	860-900	50

Relaxation times characteristic of tissue type

For materials important for human imaging
$T_1 > T_2$

Bloch equation revisited

Block equation may now be updated to include FID:

$$\frac{d\mathbf{M}}{dt} = \gamma\left(\mathbf{M} \times \mathbf{B}_0\right) - \frac{\mathbf{M}_{xy}}{T_2} + \frac{M_0 - M_z}{T_1}\hat{\mathbf{k}}$$

where:

- The first term describes the torque produced by the main (solenoid) field $\mathbf{B}_0$
- The second term describes the evolution of the transverse magnetisation vector $\mathbf{M}_{xy}$ due to the spin-spin interaction; time constant T_2
- The third term describes the evolution of the longitudinal magnetisation M_z due to the spin-lattice interaction; time constant T_1
- M_0 is the net magnetisation at equilibrium aligned with and proportional to $\mathbf{B}_0$

Complication: additional factors affecting the decay of the transverse magnetisation

T_2, the intrinsic spin-spin relaxation time is determined by non-reversible thermodynamic processes at the nuclear level.

The spin-spin time constant is reduced by a number of factors. A significant contribution comes from inhomogeneities in the main field $\mathbf{B}_0$

Inhomogeneities give rise to reversible thermodynamic processes. The associated relaxation of the transverse magnetisation is characterised by a time constant T_2'

The effective spin-spin time constant, T_2^* is given by:

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'}$$

$T_2' < T_2$ and so $T_2^* < T_2$. Need to develop techniques to recover T_2 as this carries the clinically-relevant information

Comparison of T_2 and T_2'

T_2

- The individual dipoles that sum up to produce the transverse magnetization are not precessing at precisely the same rate
- As a water molecule tumbles due to thermal motions, each H nucleus feels a small, randomly varying magnetic field in addition to B_0
- When the random field adds to B_0, the dipole precesses a little faster, and when it subtracts from B_0, it precesses a little slower
- For each nucleus the pattern of random fields is different, so as time goes on the dipoles get progressively more out of phase with one another, and as a result no longer add coherently

T_2'

- The source of this T_2' effect is magnetic field inhomogeneity
- Because the precession frequency of the local transverse magnetization is proportional to the local magnetic field, any field inhomogeneity will lead to a range of precession rates
- Over time the precessing magnetization vectors will get out of phase with one another so that they no longer add coherently to form the net magnetization
- As a result, the net signal is reduced because of this destructive interference
- Static field offsets rather than fluctuating fields

Section 2

Lecture summary

Summary

Classically, the Larmor frequency corresponds to the rate of precession of the net magnetisation vector

An RF magnetic field pulse, $\mathbf{B}_1$, oscillating at the Larmor frequency in the x, y plane is applied to rotate the net magnetisation

Free induction decay causes the longitudinal magnetisation to recover with time constant T_1 and the transverse magnetisation to decay with time constant T_2

T_1 and T_2 are characteristic of the tissue, $T_1 > T_2$

An effective spin-spin relaxation time $T_2^* < T_2$ arises due to field inhomogeneities and other effects