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Nuclear diagnostics and Magnetic Resonance Imaging

Week 2; Lecture 5; Positron Emission Tomography I

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Contents

Section 1

Principles of positron emission tomography

Positron Emission Tomography; the process

PET exploits photons generated in annihilation: $e^+ + e^- \to \gamma_1 + \gamma_2$

β^+ from decay scatters elastically off atomic electrons, loosing energy, until it annihilates

Annihilation assumed to be at rest. To conserve energy and momentum:

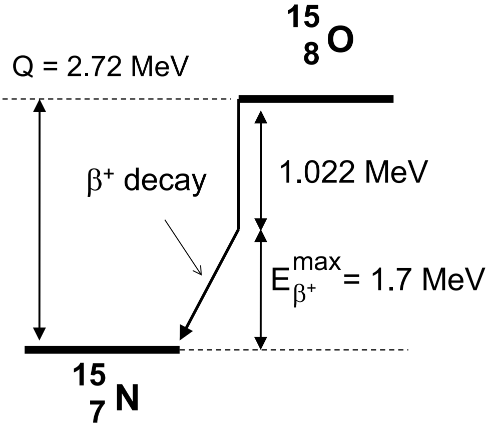
- Photons produced back-to-back
- Photon energies equal: $E_{\gamma_1} = E_{\gamma_2} = E_\gamma = mc^2 = 511\,\mathrm{keV}$

Back-to-back topology localises annihilation signal to a line in 3D space

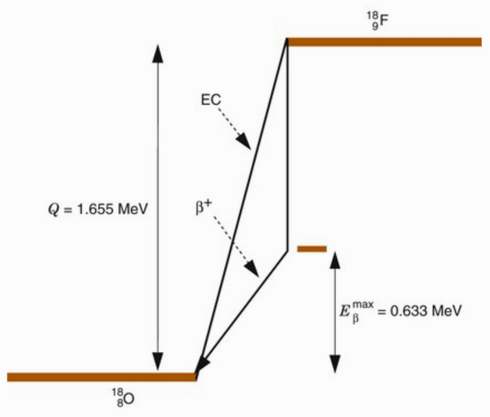
PET detectors use inorganic scintillators with large Z:

- E_γ large compared to photons used in SPECT
- So require dense scintillator with greater "stopping power" than NaI

Beta($^+$) decay, reprise by example



$$^{15}_{8}\mathrm{O} \rightarrow ^{15}_{7}\mathrm{N} + e^{-} + e^{+} + \nu_e$$

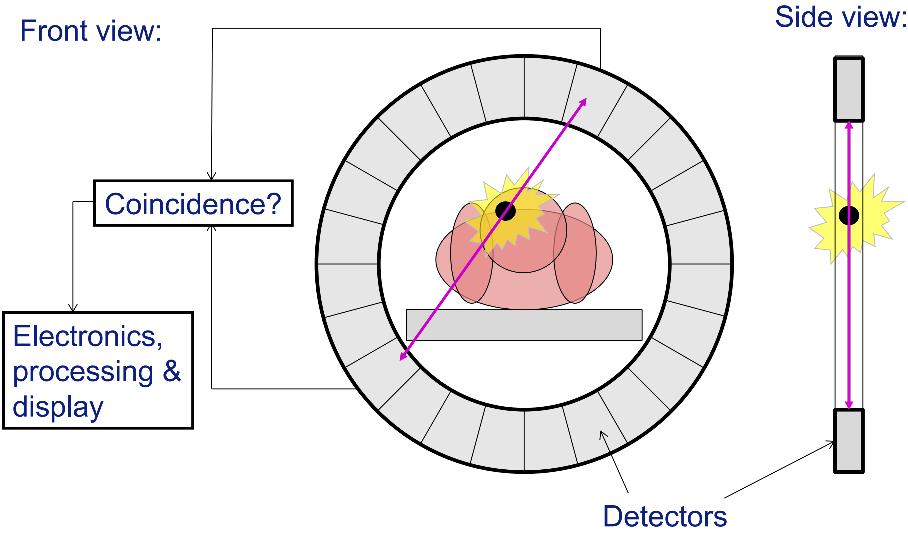


$$^{18}_{9}\mathrm{F} \rightarrow ^{18}_{8}\mathrm{O} + e^{-} + e^{+} + \nu_e$$

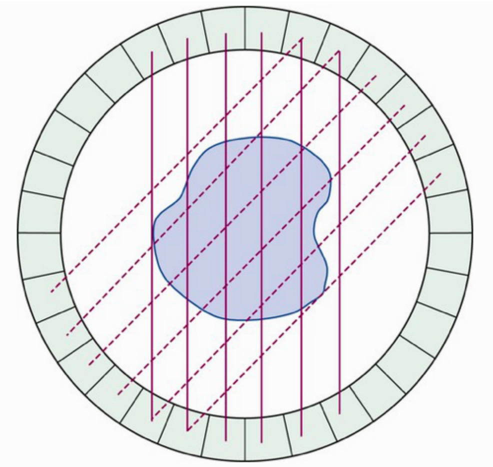
Positron emitting radionuclides

Isotope	**Half-life**	**β^+ fraction**	**Max. kinetic energy**	**Average positron range in water (mm)**
C–11	20.3 mins	0.99	0.96 MeV	1.0 mm
N–13	9.96 mins	1.00	1.19 MeV	1.3 mm
O–15	123 secs	1.00	1.72 MeV	2.0 mm
F–18	110 mins	0.97	0.64 MeV	0.6 mm
Ga–68	68.3 mins	0.88	1.90 MeV	1.2 mm
Rb–82	78 secs	0.95	3.35 MeV	2.8 mm

Principle


Front view:
Side view:
Coincidence?
Electronics, processing & display
Detectors

Taking views in parallel



Multiple projections taken at the same time:

- Schematic shows two projections
- Ring of detectors can take all projections simultaneously

  → An advantage over SPECT

"Annihilation Coincidence Detection" (ACD)

- ACD localises events to a line; "electronic collimation"

Eliminates need for absorptive septa

- Enhances geometrical efficiency substantially

  → Another advantage over SPECT

“Block detector” for PET

Cuts in scintillator:

- Do not extend to full depth
- Reflective material fills gaps

Light yield function of position

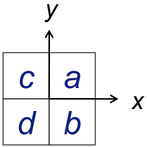
Example of “multiplexing”;
Reduces cost of optical readout

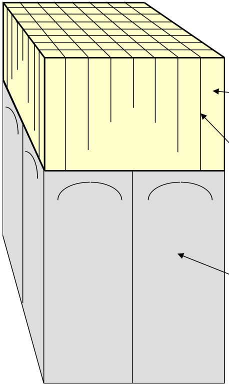
a, b, c, d are the number of counts in the 4 PMTs

ι is the size of the PMT (approximated as the side of a square)

$$X = \iota(a + b - c - d) / (a + b + c + d)$$

$$Y = \iota(a - b + c - d) / (a + b + c + d)$$





Segmented block of BGO or LSO scintillator

Reflector material between segments

Four single channel photomultiplier tubes

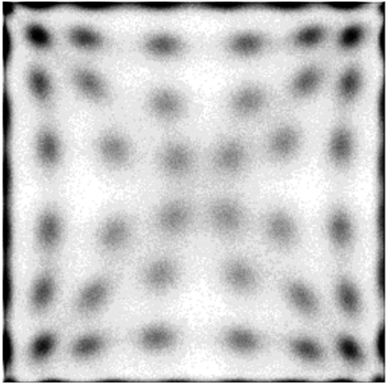
N.B. in the formula on this slide, *X* and *Y* are measured relative to the *centre* of block of 4 PMTs

Example:
64 crystals, 4 pmts:
multiplexing factor of 16

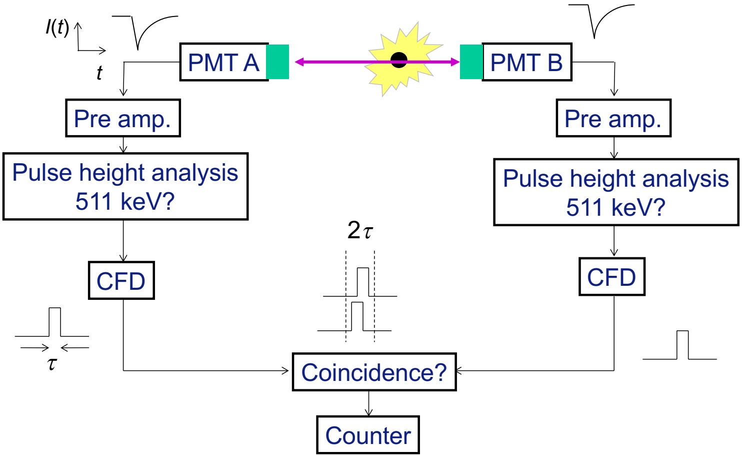
"Block detector" for PET

Flood irradiation of block detector with 511 keV γs:

- Spatial localisation of energy deposits
- Non-linear response corrected with "look-up table"

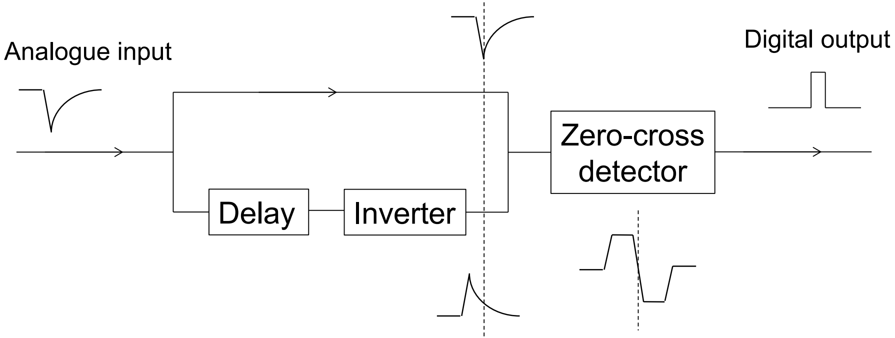


Forming the coincidence



CFD: constant fraction discriminator

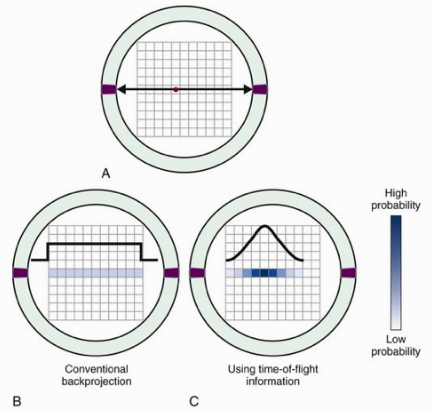
Constant fraction discriminator (CFD)



Objective: determine arrival time of pulse that is largely independent of pulse height

- Time at which signal reaches a fraction (e.g. 25%) of its peak amplitude

Time-of-flight measurement



A: ACD configuration
B: No ToF: back-projection with equal probability
C: ToF: back-projection localised at Δd

If time-of-arrival difference is Δt, then:

$$\Delta d = \frac{c\Delta t}{2}$$

where Δd is measured w.r.t. the midpoint

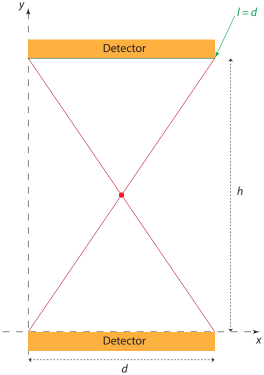
Localises back projection to $\Delta d \pm \sigma_{\Delta d}$

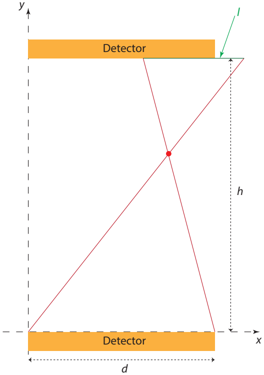
Requires fast scintillator, fast electronics to yield $\Delta t \sim 100 - 200\,\mathrm{ps}$

Leads to $\sigma_{\Delta d} \sim 2 - 3\,\mathrm{cm}$

Of benefit in removal of ambiguities in reconstruction

Intrinsic spatial resolution

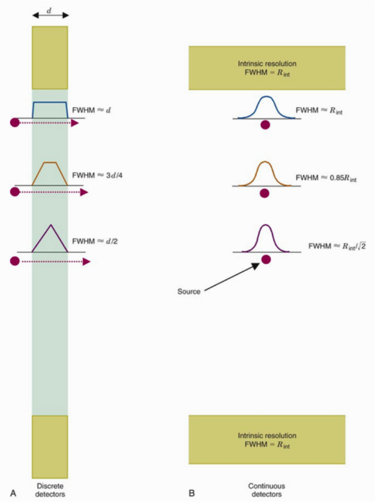




Source at $\left(x = \frac{d}{2}, y = \frac{d}{2}\right)$;
all events in cone accepted

Source at (x, y);
accept only events striking both detectors

Intrinsic spatial resolution



With coordinates defined above, projected length l is given by:

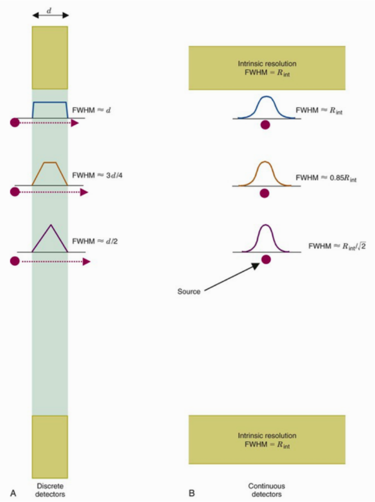
$$l = d\frac{h-y}{y}$$

Observations:

- At fixed y, l is independent of x
- Intensity recorded is a function of x
- At fixed y:
  - PDF rises from 0 at $x = 0$ and $x = d$
  - Plateau in PDF reached when $x_{\mathrm{P}} = \frac{d}{h}y$
- For $y = \frac{h}{2}$ no plateau because $x_{\mathrm{P}} = \frac{d}{2}$

Shown in LH column of figure

Intrinsic spatial resolution



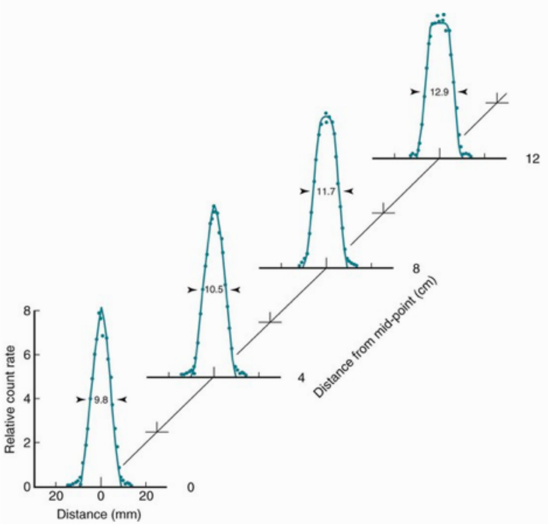
Referring now to RH column of figure . . .

Define FWHM at $y = h$: $\mathrm{FWHM} = R_{\mathrm{int}}$

Intrinsic resolution:

- $y = h$: rectangular distribution
  $$\sigma_{\mathrm{int}} = \frac{R_{\mathrm{int}}}{2} = \frac{d}{\sqrt{12}}$$
- $y = \frac{h}{2}$: triangular distribution
  $$\sigma_{\mathrm{int}} = \frac{R_{\mathrm{int}}}{2\sqrt{2}} = \frac{d}{\sqrt{2}\sqrt{12}}$$
- Intermediate y: "truncated triangle"
  $$\frac{d}{\sqrt{2}\sqrt{12}} \leq \sigma_{\mathrm{int}} \leq \frac{d}{\sqrt{12}}$$

Measured intrinsic resolution



Measured "residuals" for:

- 2 detectors each with $d = 17\,\text{mm}$
- $\sigma_{\rm int} = 4.91\,\text{mm}$

  $$\Rightarrow \mathrm{FWHM} = R_{\rm int} = 9.81\,\text{mm}$$

Resolution favourable cf SPECT with:

- Conjugate sampling
- Arithmetic mean position estimation

For equivalent position resolution PET is:

- More efficient that SPECT (collimator)
- Faster; all images taken at once

Summary of section 1

PET exploits kinematics of $e^{+}e-$ annihilation using coincident, conjugate detection to localise position of a particular disintegration to a line

Appropriate readout electronics allows multiple projections to be acquired at the same time; an improvement in efficiency compared to SPECT

Time-of-flight measurement used to localise position of disintegration along line defined annihilation-photon pair

Spatial resolution varies slowly across fiducial volume

Section 2

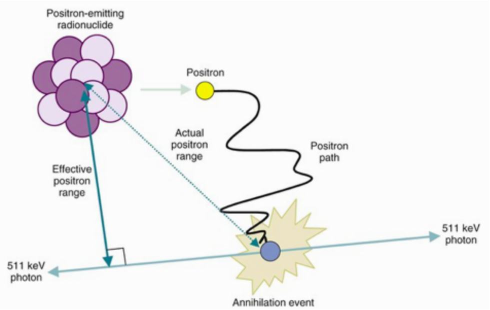
System resolution

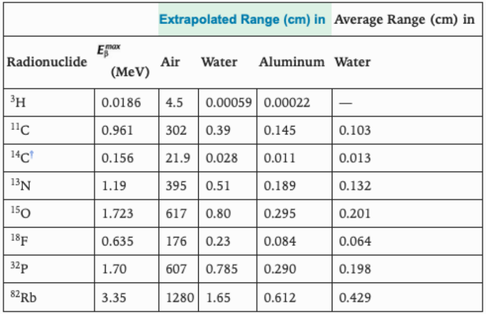
Factors that determine system resolution

Intrinsic resolution of PET system degraded by:

- Fundamental physics:
    - Non-zero range of positron as it slows down prior to annihilation
    - Residual momentum of positron at annihilation results in non-colinear photons
- Reconstruction:
    - Depth-of-interaction effect
    - Sampling effect
    - Filter effect

Fundamental physics: positron range





		Extrapolated Range (cm) in			Average Range (cm) in
Radionuclide	E_β^{max} (MeV)	Air	Water	Aluminum	Water
^{3}H	0.0186	4.5	0.00059	0.00022	—
^{11}C	0.961	302	0.39	0.145	0.103
^{14}C†	0.156	21.9	0.028	0.011	0.013
^{13}N	1.19	395	0.51	0.189	0.132
^{15}O	1.723	617	0.80	0.295	0.201
^{18}F	0.635	176	0.23	0.084	0.064
^{32}P	1.70	607	0.785	0.290	0.198
^{82}Rb	3.35	1280	1.65	0.612	0.429

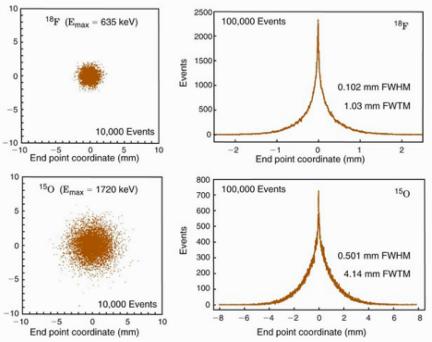
Effective range depends on:

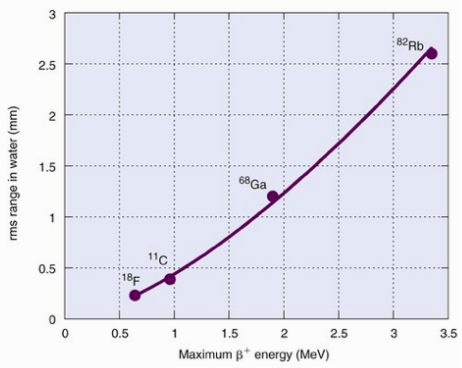
- $E_{\max}$
- Material, i.e. tissue

For relevant PET isotopes:

- $E_{\max}$ in range 0.5–1.8 MeV
- Results in e^+ range in range 2–8 mm

Fundamental physics: positron range





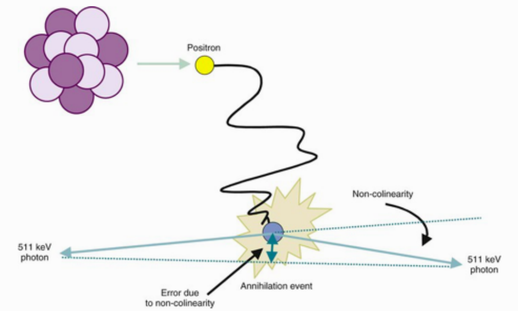
End-point coordinate distribution falls exponentially; certainly not Gaussian!

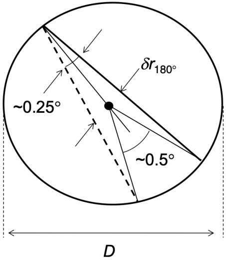
RMS of the effective range distribution used:
FWHM: $R_{\rm range}$; resolution: $\sigma_{\rm range}$

Resolution improves as $E_{\rm max}$ falls

- ^{18}F gives improved resolution over other commonly used isotopes

Fundamental physics: non-colinearity





Non-colinearity angular distribution:

- Sufficiently Gaussian, use FWHM
- FWHM approximately 0.5°

Geometrically:

$$R_{180} = \frac{D}{2} \times 0.25 \frac{\pi}{180} = 0.0022 \times D$$

and so resolution is $\sigma_{180} = \frac{R_{180}}{2}$

System resolution

System resolution, taken to be the resolution of the hardware, may now be evaluated:

- In terms of FWHM:

$$R_{\rm sys} = \sqrt{R^2_{\rm int} + R^2_{\rm range} + R^2_{180}}$$

- In terms of resolution:

$$\sigma_{\rm sys} = \sqrt{\sigma^2_{\rm int} + \sigma^2_{\rm range} + \sigma^2_{180}}$$

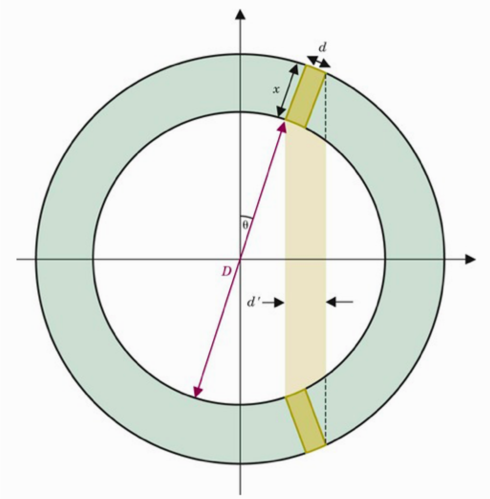
Example: clinical PET scanner:

- 5 mm scintillator: $R_{\rm int} = 2.5$ mm
- ^{18}F-labelled tracer: $R_{\rm range} = 0.6$ mm
- 800 mm diameter scanner: $R_{180} = 1.8$ mm

Yields:

- $R_{\rm sys} = 3.1$ mm

Reconstruction: depth-of-interaction (DOI) effect



Thickness of scintillator used to stop 511 keV γ introduces a reconstruction uncertainty

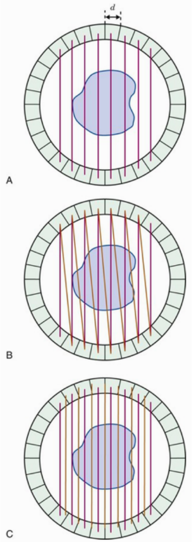
For the case sketched, the apparent width of the detector, d', is given by:

$$d' = d\cos\theta + x\sin\theta$$

Can now use d' in the formulæ for, e.g., $R_{\rm int}$

In a typical system, the DOI effect causes a degradation of $\sim 40\%$ in the resolution at a distance of 100 mm from the centre

Reconstruction: sampling effect



The intrinsic resolution is determined by the detector size, *d*

A) Sampling "frequency" determined by spacing, also *d*
Limits minimum feature size that can be resolved

B) Record neighbouring coincidences
Improved sampling; can reduce minimum feature size

c) Treat "neighbouring coincidences" (B) as additional samples
Implementation leads to improvement in detail in image

Reconstruction: filter effect

Image reconstruction exploits techniques such as filtered back projection

Filters are used to suppress noise, but, removing frequencies from the Fourier transform of the image can also remove detail from the image

Image-processing strategies need to be tailored to the situation, e.g. brain scans may require different strategies to abdominal scans

Summary of section 2

Overall resolution of PET system determined by:

- “Physics”:
    - Positron range;
    - Positron momentum at annihilation ⇒ photon non-colinearity
- “Reconstruction”:
    - Depth-of-interaction (DOI) effect;
    - Sampling effect
    - Filter effect

Section 3

Sensitivity

Sensitivity

Sensitivity is determined primarily by detector efficiency and solid angle coverage

True coincidence count rate $\mathcal{R}_{\mathrm{True}}$ for a positron-emitting source in air near midpoint between a pair of detectors is:

$$\mathcal{R}_{\mathrm{True}} = \left(\mathcal{R}_{\mathrm{e^+}}\right) \epsilon^2 G \exp\left(-\mu T\right)$$

where:

- $\mathcal{R}_{\mathrm{e^+}}$ is the rate of positron emission (positrons/sec)
- ϵ is the intrinsic detector efficiency:

$$\epsilon = \frac{\text{no. of } \gamma\text{−rays recorded by detector}}{\text{no. of } \gamma\text{−rays ``hitting'' detector}}$$

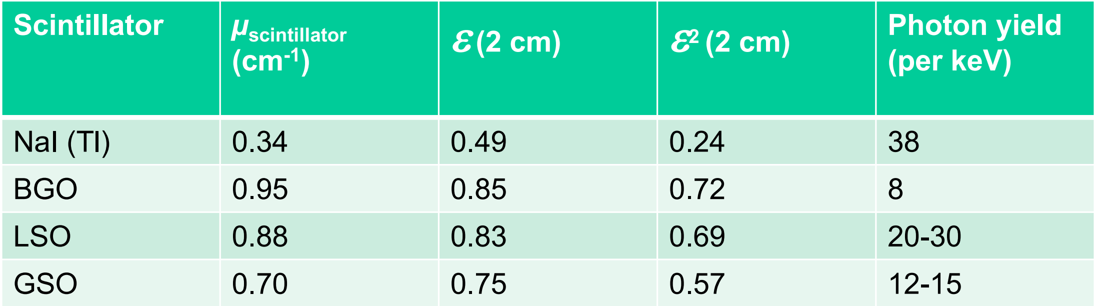
- G is the geometric efficiency of an individual detector:

$$G \approx \frac{A_{\mathrm{det}}}{\pi D^2}$$

- μ is the linear attenuation coefficient, T the total thickness

Sensitivity: examples of efficiencies

Intrinsic detector efficiency for a variety of scintillators



Scintillator	$\mu_{\text{scintillator}}$ (cm^{-1})	ε(2 cm)	ε^2 (2 cm)	Photon yield (per keV)
NaI (Tl)	0.34	0.49	0.24	38
BGO	0.95	0.85	0.72	8
LSO	0.88	0.83	0.69	20-30
GSO	0.70	0.75	0.57	12-15

Summary of section 3

Sensitivity determined by:

- Activity of the radiotracer;
- Intrinsic efficiency of the detector;
- Geometrical efficiency of the detector; and
- Containment of energy deposit in scintillator crystal