

# Nuclear diagnostics and Magnetic Resonance Imaging

## Week 3; Lecture 7; Introduction to MRI and quantum-mechanical foundations

K. Long ([k.long@imperial.ac.uk](mailto:k.long@imperial.ac.uk))

Department of Physics, Imperial College London/STFC

R. McLauchlan ([ruth.mclauchlan@nhs.net](mailto:ruth.mclauchlan@nhs.net))

Radiation Physics & Radiobiology Department, Imperial College Healthcare NHS Trust

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## Section 1

## Introduction to MRI

# 'Guilt-free' imaging



Whole-body imager, Star Trek style

Nuclear diagnostics and X-ray imaging:

- Image constructed using ionising radiation
- Necessarily delivers dose to patient
- Dose implies risk of initiating disease

Magnetic resonance imaging (MRI):

- Image generated by exploiting magnetic moment of H nuclei
- Patient immersed in magnetic field
- No permanent harmful effects reported

# Nuclear magnetic moment

Proton (and neutron) magnetic moment:

- Nucleons each have spin of  $\frac{1}{2}$
- Magnetic moment generated by nuclear charge
  - Contributions to nuclear spin arise from quarks and gluons. Quantitative explanation of nuclear magnetic moment is an active area of research
- For NMR and MRI critical point is that the magnetic moment,  $\mu$ , is related to the nuclear spin,  $\mathbf{s}$  by:

$$\mu = \gamma \mathbf{s}$$

where  $\gamma$  is the “gyromagnetic” ratio

# Nuclear magnetic resonance

Effect of uniform magnetic field **B**:

- **B** provides “quantisation axis”:  
    ⇒ nuclear dipoles align with magnetic field
- For proton spin is  $\frac{1}{2}$ , so only two states:  
    Spin “up” and spin “down”
- Energy splitting; 2 energy levels:
  - Lower energy level has magnetic moment parallel to magnetic field
  - Higher energy level has magnetic moment anti-parallel to magnetic field
- Resonance:
  - Call energy splitting  $\Delta E$
  - Transitions between the two energy levels cause absorption or emission of electromagnetic (em) radiation for which  $\Delta E = h\nu$
  - Resonance occurs when em radiation of frequency  $\nu$  is injected

# Magnetic resonance imaging

Magnetic resonance imaging (MRI) exploits this resonance

Steps:

- Apply uniform magnetic field, align proton ( $^1\text{H}$ ) spins
- Apply radiation, at exactly  $\nu$ , to cause transitions between “spin up” & “spin down” states
- Turn off the radiation ... and ...
- “Listen” for radiation at exactly  $\nu$  as the spins realign

Brilliant! Simple principle and elegant technique. Now exploited in exquisitely sophisticated imaging systems.

# The physical principles

**1938: I. Rabi:** Discovered nuclear magnetic resonance

Nobel Prize 1944

**1946: F. Bloch & E. Purcell:** Developed methods that allow precision methods using NMR

Nobel Prize 1952

**1955/56: E. Odeblad & G. Lindström:** Applied NMR to living cells from animal tissue

**1968: J.A. Jackson and W.H. Langham:** First NMR measurements from living animals

# Cancerous and normal cells differ



Relaxation times that characterise recovery of ground-state magnetisation shown to differ between normal and tumour cells

Raymond Damadian

Tumor Detection by Nuclear Magnetic Resonance

Author(s): Raymond Damadian

Source: *Science*, New Series, Vol. 171, No. 3976 (Mar. 19, 1971), pp. 1151-1153

Published by: American Association for the Advancement of Science

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Accessed: 01-03-2020 09:22 UTC

## REFERENCES

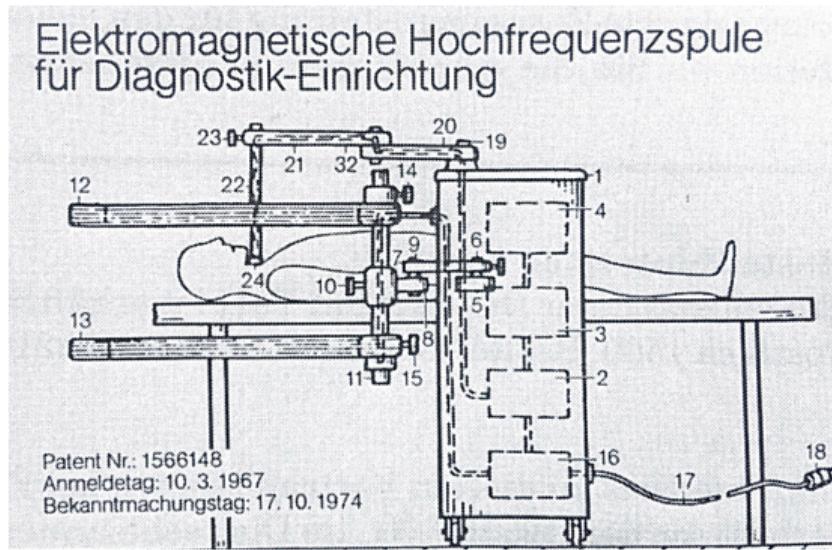
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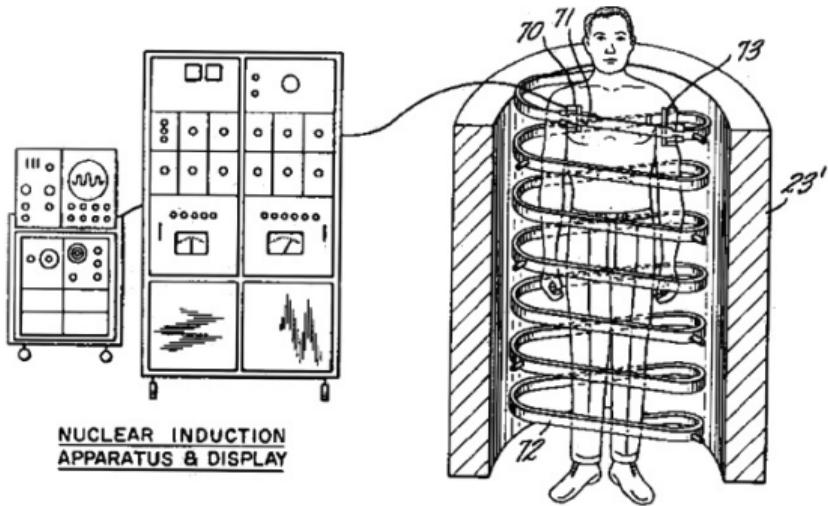
You may need to log in to JSTOR to access the linked references.

# Early proposals for MRI scanners

Alexander Ganssen; patent 1967



Raymond Damadian; patent 1972



# Spatial localisation using magnetic-field gradients



Superimpose field gradient on main uniform magnetic field. Incident em radiation at frequency  $\nu$  only resident in a particular location in subject

Paul Lauterbur

Nature Vol. 242 16 March 1973

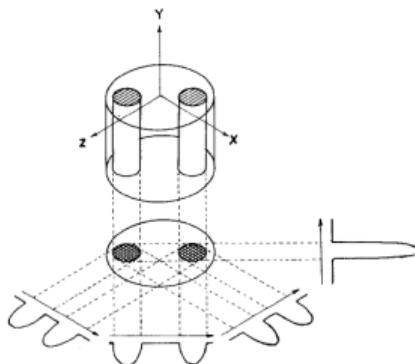


Fig. 1 Relationship between a three-dimensional object, its two-dimensional projection along the Y-axis, and four one-dimensional projections at 45° intervals in the XZ-plane. The arrows indicate the gradient directions.



Fig. 2 Proton nuclear magnetic resonance zeugmatogram of the object described in the text, using four relative orientations of object and gradients as diagrammed in Fig. 1.

# Rapid, “snap-shot” MRI



Use of “echo planar imaging” to allow fast “snap-shot” imaging required active screening of fields created by currents induced in cryostat walls

Peter Mansfield

P. Mansfield, Nobel Lecture 2003



Figure 2. Photograph of a doubly screened active magnetic shielded gradient coil set for insertion in the super-conductive magnet of Figure 1.

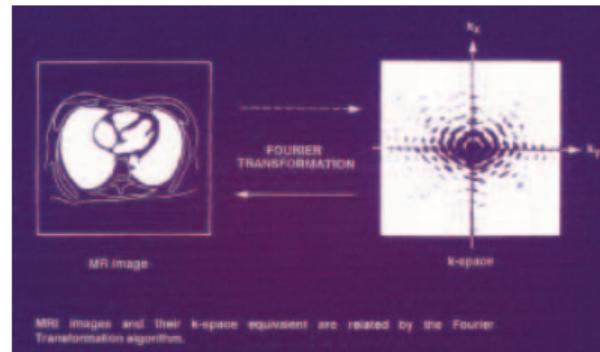


Figure 3. Diagram of a slice through the mediastinum showing the two lung fields and heart mass, also shown is the Fourier transform of this real-space image to the k-space map. (Reproduced with permission from M K Stehling, R Turner and P Mansfield, SCIENCE 253, 43–50 (1991).)

# NMR zeugmatography

**1975:** A. Kumar, D. Welti, R. Ernst

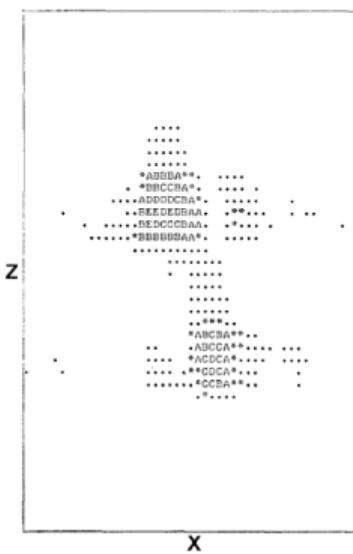
NMR FOURIER ZEUGMATOGRAPHY

Application of Fourier techniques to the reconstruction of images

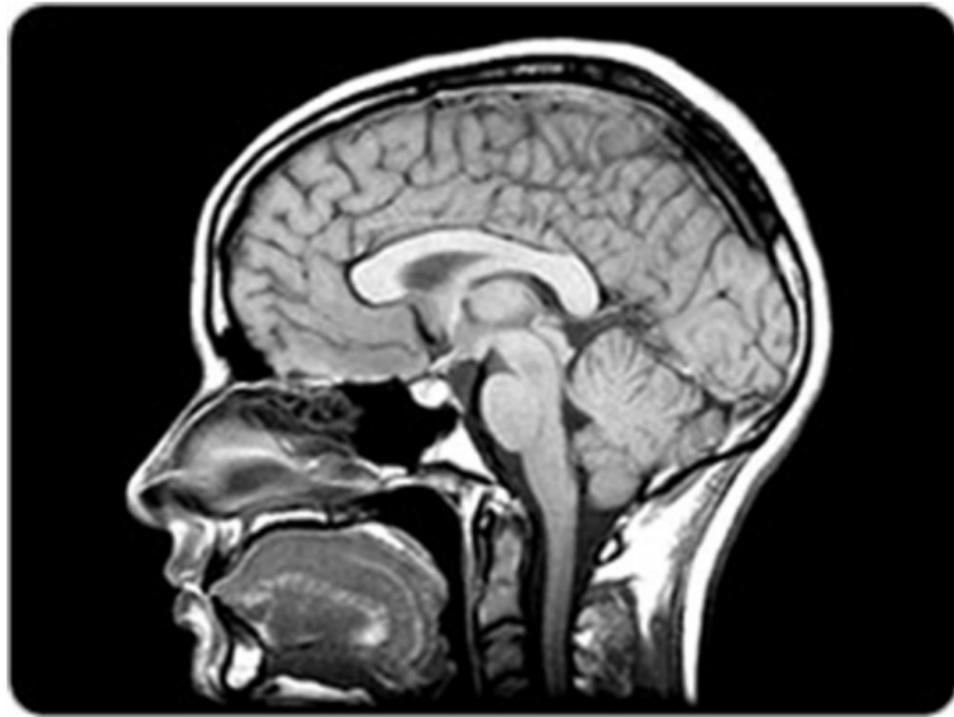
Journal of Magnetic Resonance, Vol 18, P 69-83(1975)

**zeug·ma·tog·ra·phy** (zūg'mă-tog'ră-fē),

Term coined by Lauterbur in 1972 for the joining of a magnetic field and spatially defined radiofrequency field gradients to generate a two-dimensional display of proton density and relaxation times in tissues, the first nuclear magnetic resonance image.



# State of the art



## Summary of section 1

Magnetic moment of proton exploited to provide energy splitting,  $\Delta E$  between spin-up and spin-down states in applied magnetic field

Injection of radio-frequency wave with a frequency that resonates with the splitting then used to manipulate population of protons in the spin-up and spin down states

Images produced by manipulating applied magnetic field and frequency of RF field gradients

## Section 2

# Quantum mechanical foundations

# Theoretical description; a hybrid of quantum and classical

Nuclear magnetic resonance & MRI are both inherently quantum mechanical effects:

- Signal is generated by manipulating the *spins* of hydrogen nuclei:
  - Spin is postulated to explain hyperfine structure, Stern-Gerlach experiment, ...
  - Understood theoretically through the symmetries of space and time
- Magnetic moment of proton,  $\mu$ , is related to the proton spin,  $s$ , by:

$$\mu = \gamma s$$

where  $\gamma$  is the “gyromagnetic ratio”

Hybrid, quantum/classical treatment:

- Quantum mechanics: energy splitting and population in ground and excited state
- Classical: magnetisation vector, its precession, and the manipulation of the magnetisation vector to generate the signals used for imaging

## Interaction of nuclear magnetic dipole with uniform magnetic field

The contribution,  $\delta\mathcal{U}$ , to the potential energy of a proton immersed in a magnetic field,  $\mathbf{B}$ , is given by:

$$\delta\mathcal{U} = -\mathbf{B} \cdot \boldsymbol{\mu}$$

Lets consider a proton which, in the absence of a magnetic field has energy  $E$ . Applying the magnetic field introduces  $\delta\mathcal{U}$  into the Schrödinger equation resulting in a splitting of the proton energy level such that  $E \rightarrow E'$  given by:

$$E' = E \pm E_{m_s}$$

where

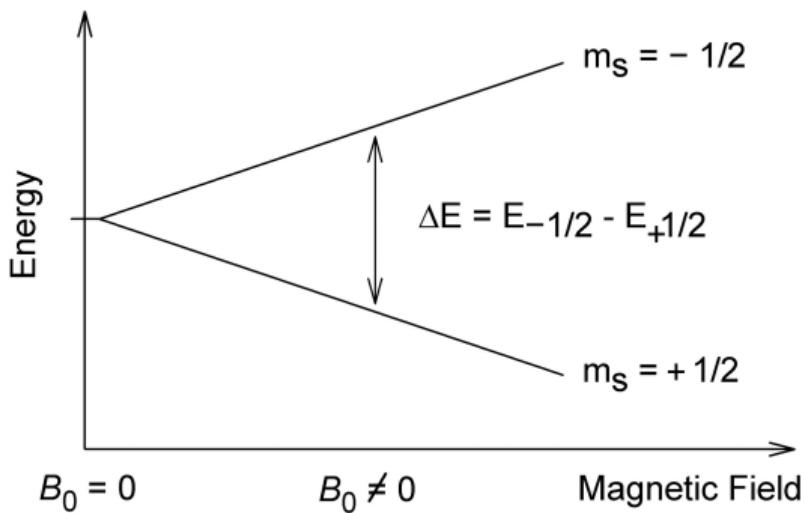
$$E_{m_s} = -m_s \gamma \hbar B_0$$

where  $m_s$  is the quantum number associated with the component of the proton spin parallel to  $\mathbf{B}$ ,  $\hbar$  is Planck's constant divided by  $2\pi$ , and  $B_0$  is the magnitude of  $\mathbf{B}$

For the proton:

$$m_s = \pm \frac{1}{2}$$

# Larmor equation



$\Delta E$ , splitting between two levels with  $m_s = \pm \frac{1}{2}$ :

$$\Delta E = \gamma \hbar B_0$$

Planck's law relates energy splitting to the angular frequency,  $\omega$ , of the radiation required to excite the transition, therefore:

$$\Delta E = \hbar \omega$$

Writing  $\omega$  in terms of  $\gamma$  and  $B_0$  yields the Larmor equation:

$$\omega = \gamma B_0$$

# Gyromagnetic ratios of some nuclei

Definition of gyromagnetic ratio,  $\gamma$ :

*The gyromagnetic ratio,  $\gamma$ , of a particle or system is the ratio of its magnetic dipole moment to its angular momentum*

For charged body of charge  $q$ , mass  $m$  rotating about an axis of symmetry:

$$\gamma = \frac{qe}{2m}$$

where  $e$  is the magnitude of the charge on the electron

For proton,  $q = 1$ ,  $m = m_p$ , the proton mass.

$\gamma$  is sometimes used instead of  $\gamma$ :

$$\gamma = \frac{\gamma}{2\pi}$$

nucleus	$\gamma$ (rad MHz T <sup>-1</sup> )	$\gamma = \gamma / 2\pi$
<sup>1</sup> H	267.513	42.576
<sup>2</sup> H	41.065	6.536
<sup>3</sup> He	203.789	32.434
<sup>7</sup> Li	103.962	16.546
<sup>13</sup> C	67.262	10.705
<sup>14</sup> N	19.331	3.077
<sup>15</sup> N	27.116	-4.316
<sup>17</sup> O	36.264	5.772
<sup>19</sup> F	251.662	40.053
<sup>23</sup> Na	70.761	11.262
<sup>27</sup> Al	69.763	11.103
<sup>31</sup> P	108.291	17.235
<sup>57</sup> Fe	8.681	1.382
<sup>63</sup> Cu	71.118	11.319
<sup>67</sup> Zn	16.767	2.669
<sup>129</sup> Xe	73.997	11.777

# Examples

Larmor equation:  $\omega = \gamma B_0 \Rightarrow \nu = \gamma B_0$

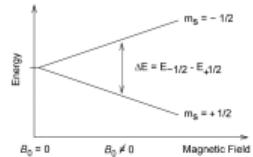
For hydrogen nucleus,  ${}^1\text{H}$ ,  $\gamma = 42.58 \text{ MHz/T}$

What is the resonance frequency for  ${}^1\text{H}$  when:

- $B_0 = 1.5 \text{ T}$ ?
- $B_0 = 3.0 \text{ T}$ ?

What are the corresponding values for the energy splittings  $\Delta E = h\nu$ , where  $h$  is Planck's constant?

## Populations in the two spin states



$^1\text{H}$  in tissue in thermal equilibrium, so, partition between the populations in the two spin states follows the Boltzmann distribution:

$$\frac{N_+}{N_-} = \exp\left(-\frac{\Delta E}{k_B T}\right)$$

where  $N_+$  and  $N_-$  are the number of  $^1\text{H}$  in  $+\Delta E$  and  $-\Delta E$  states respectively,  $k_B$  is Boltzmann's constant, and  $T$  is the temperature

For the human body,  $k_B T \approx 25.7 \text{ meV}$ , so:

$$\Delta E \ll k_B T$$

Therefore, expanding the exponential and rearranging:

$$N_- - N_+ \approx N_S \frac{\Delta E}{2k_B T}$$

# Magnetisation

Substituting for  $\Delta E$

$$N_- - N_+ \approx N_S \frac{\Delta E}{2k_B T} = N_S \frac{\gamma h B_0}{4\pi k_B T}$$

For  $B_0 = 1.5 \text{ T}$ :

$$\begin{aligned}\frac{N_- - N_+}{N_S} &\approx \frac{42.58 \times 10^6 \times 6.6 \times 10^{-34} \times 1.5}{2 \times 1.38 \times 10^{-23} \times 300} \\ &\approx 4.5 \times 10^{-6}\end{aligned}$$

i.e. only 4.5 in a million protons in the body are available for activation in MRI at  $B_0 = 1.5 \text{ T}$

## Bulk magnetisation is measurable

Population-density “mismatch” of  $\approx 3 \text{ ppm}$  per Tesla arises due to fact that energy splitting is small compared to  $k_B T$

Bulk magnetisation still measurable because 1 gram of water contains  $10^{22} {}^1\text{H}$

## Summary of section 2

Larmor frequency,  $\omega$  determined by the gyromagnetic ratio,  $\gamma$ , and the applied magnetic field,  $B_0$ :  $\omega = \gamma B_0$

In presence of  $B_0$  at temperature  $T$  the equilibrium magnetisation of a sample of hydrogen nuclei is small, but measurable, and aligned with the applied magnetic field