Centre for the Clinical Application of Particles

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Magnetic Resonance Imaging

Week 4; Lecture 8; Section 1: Classical derivation of the Larmor equation

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Section 1

Classical derivation of Larmor equation

The Larmor equation and bulk magnetisation; reprise

The quantum mechanical treatment presented in lecture 7 led to the **Larmor equation**:

$$\omega = \gamma B_0$$

ω is the Larmor frequency, B_0 the magnitude of the magnetic field, γ the gyromagnetic ratio

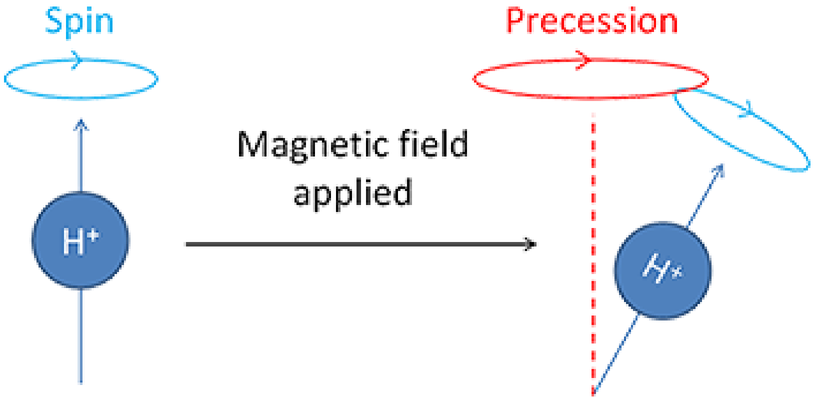
ω is the resonant frequency in an external magnetic field

The **bulk magnetisation** was obtained by considering the partition between the two energy states of the ^{1}H nuclei in the magnetic field:

$$N_- - N_+ \approx N_S \frac{\Delta E}{2k_\mathrm{B}T} = N_S \frac{\gamma \hbar B_0}{2k_\mathrm{B}T}$$

where the notation is that defined in lecture 7

Classical magnetic moment in magnetic field

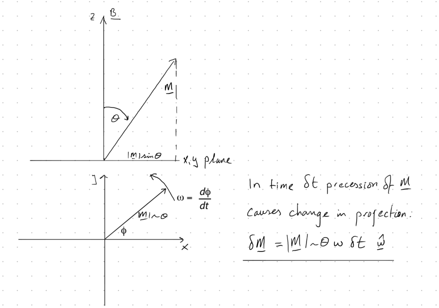


Magnetic moment that makes an angle with a magnetic field will precess around the magnetic-field axis.

Classical derivation of the Larmor equation

Classically, a magnetic moment, **M**, in a magnetic field **B**, experiences a torque given by the Bloch equation:

$$\frac{d\mathbf{M}}{dt} = \gamma\left(\mathbf{M} \times \mathbf{B}\right)$$



M makes an angle θ w.r.t. **B**. So:

$$\mathbf{M} \times \mathbf{B} = MB_0 \sin\theta\hat{\boldsymbol{\omega}}$$

So:

$$\frac{d\mathbf{M}}{dt} = \gamma MB_0 \sin\theta\hat{\boldsymbol{\omega}} = M \sin\theta\omega\hat{\boldsymbol{\omega}}$$

Which gives the Larmor equation:

$$\omega = \gamma B_0$$

Examples

Larmor equation: $\omega = \gamma B_0 \quad \Rightarrow \quad \nu = \gamma\!\!\!-\! B_0$

Energy splitting: $\Delta E = \hbar\omega \quad \Rightarrow \quad \Delta E = h\nu$

$h = 4.1357 \times 10^{-15}\,\mathrm{eV\,s}$

For hydrogen nucleus, $^{1}\mathrm{H}$, $\gamma\!\!\!-\! = 42.58\,\mathrm{MHz/T}$

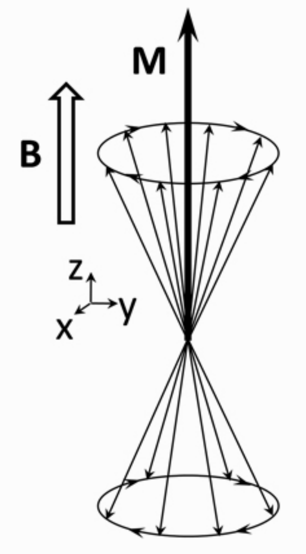
Calculating the values of ν and ΔE yields:

Magnetic field B_0 (T)	Larmor frequency (MHz)	ΔE (eV)
1.5	63.87	2.64E-07
3.0	127.74	5.28E-07

For comparison:

- FM radio waveband runs from 88.1 MHz to 108.1 MHz;
- $k_{\mathrm{B}} T = 2.59 \times 10^{-2}\,\mathrm{eV}$

Larmor precession



Ensemble of ^{1}H nuclei, the majority (by $\approx 3\,\mathrm{ppm\,T^{-1}}$) orientated parallel to **B** precess at equilibrium around **B** at the Larmor angular frequency ω

Net magnetisation, **M**, produced is parallel to **B**.

There is no net magnetisation in the transverse (x, y) plane; sum of all contributions cancel

Result is that there is no change in the magnitude or direction of the magnetisation vector so no RF signal is produced

Key feature of MRI: manipulate **M** so as to produce a measurable RF signal

Summary of section 1

Magnetisation vector, **M**, created by unequal number of ^{1}H spins parallel and anti-parallel to the applied magnetic field **B**

The magnetisation vector precesses around the direction defined by the applied magnetic field at the Larmor frequency, ω

The Larmor frequency is given by:

$$\omega = \gamma B_0$$

This is the same Larmor frequency that was obtained in the quantum-mechanical discussion of the splitting of the energy level of the ^{1}H nucleus