

Magnetic Resonance Imaging

Week 6; Lecture 12; Section 2: Wraparound artefact and the Gibbs phenomenon

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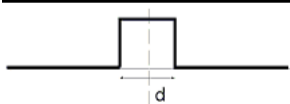
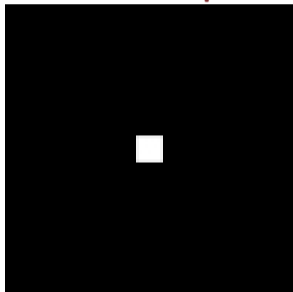
Section 2

Aliasing (wraparound) artefact and the Gibbs phenomenon

Trial: square centred at $(x, y) = (0, 0)$

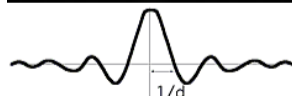
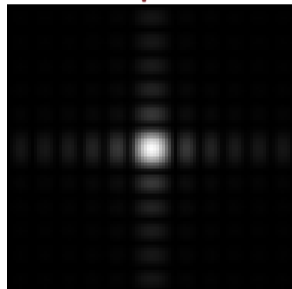
Consider spatial encoding, x, direction

Coordinate space



$$\rho(x) = \begin{cases} \rho_0 & \text{for } -\frac{d}{2} < x < \frac{d}{2} \\ 0 & \text{otherwise} \end{cases}$$

k space



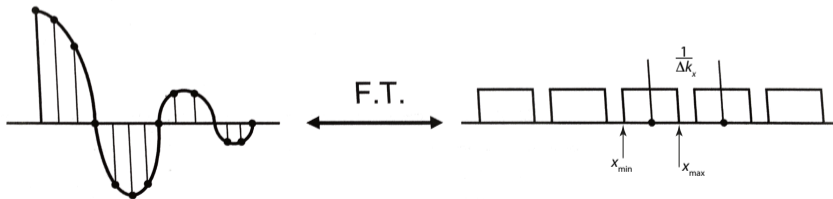
$$S(\Delta k_x) = S'_0 \text{sinc}(\Delta k_x)$$

Sampling of the signal recorded along k_x

The Fourier transform of “sinc” function will give “box” function if all Δk_x are sampled

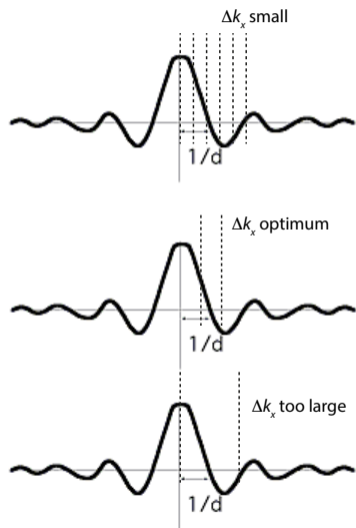
But, sinc function is only sampled at intervals of Δk_x

This means that Fourier transform of the sampled sinc function generates a series of (distorted) images of the box:



Sequence truncated at field of view

Sampling of the signal recorded along k_x



Nyquist theorem:

To reconstruct a bandwidth limited signal, require to sample the highest frequency that the signal contains at least twice

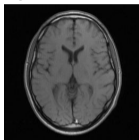
In our case, the bandwidth is limited by the truncation of the sinc function by the field of view in k space

At limit of resolution, box functions are “just separated”

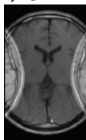
If sampling rate is too low, $\Delta k_x > \frac{2}{2\pi(x_{\max} - x_{\min})}$, the boxes overlap and aliasing occurs

Aliasing

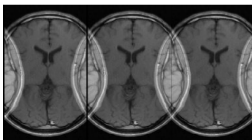
Image of head with appropriate sampling rate (field of view)



Truncated field of view (in k -space) yields sampling rate that is too low ...

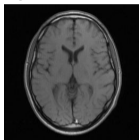


... and leads to aliasing

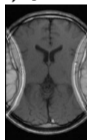


Aliasing (wraparound)

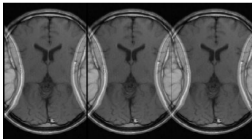
Image of head with appropriate sampling rate (field of view)



Truncated field of view (in k -space) yields sampling rate that is too low ...



... and leads to aliasing

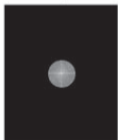


Effect of truncation in k space

Normal Lincoln



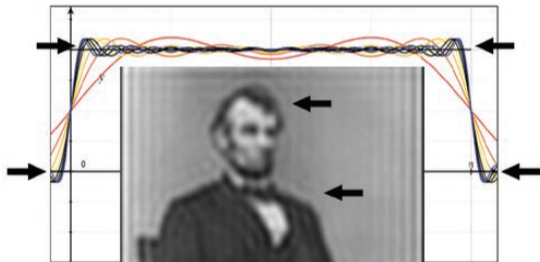
FT

High Frequencies
Removed

Inverse FT



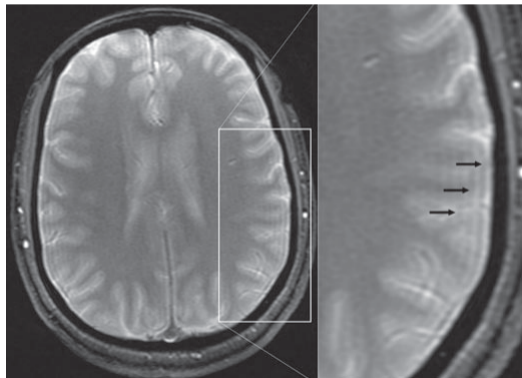
Blurry Lincoln



The Gibbs phenomenon

Artefact occurs at interfaces between tissues which have a rapid change in signal ... "high contrast interfaces"

E.g. skull to brain



Summary of section 2

Nyquist theorem:

For accurate reconstruction of features require to sample the highest frequency at least twice

If the sampling rate is too low, aliasing (or wraparound) artefacts occur

Truncation of sinc function by field of view in k -space can lead to image blur and the “Gibbs phenomenon” at places where there is a rapid change in image brightness