Centre for the Clinical Application of Particles

January 1, 2022

Physics of Medical Imaging and Radiotherapy

Magnetic Resonance Imaging

Lecture 1; Section 1: Introduction to MRI

K. Long (k.long@imperial.ac.uk)
Department of Physics, Imperial College London/STFC

Section 1

Introduction to MRI

‘Guilt-free’ imaging

Whole-body imager, Star Trek style

Nuclear diagnostics and X-ray imaging:

- Image constructed using ionising radiation
- Necessarily delivers dose to patient
- Dose implies risk of initiating disease

Magnetic resonance imaging (MRI):

- Image generated by exploiting magnetic moment of H nuclei
- Patient immersed in magnetic field
- No permanent harmful effects reported

Nuclear magnetic moment

Proton (and neutron) magnetic moment:

- Nucleons each have spin $\frac{1}{2}$
- Magnetic moment generated by nuclear charge
  - Contributions to nuclear spin arise from quarks and gluons
  - Quantitative explanation of nuclear magnetic moment is active field of research
- Magnetic moment, $\boldsymbol{\mu}$, is related to nuclear spin, $\mathbf{s}$ by:

$$\boldsymbol{\mu} = \gamma \mathbf{s}$$

where γ is the "gyromagnetic" ratio

Nuclear magnetic resonance

Effect of uniform magnetic field **B**:

- **B** provides "quantisation axis":
  $\Rightarrow$ nuclear dipoles align with magnetic field
- Proton spin is $\frac{1}{2}$, so only two states:
  Spin "up" and spin "down"
- Energy splitting; 2 energy levels:
  - Lower energy level has magnetic moment parallel to magnetic field
  - Higher energy level has magnetic moment anti-parallel to magnetic field
- Resonance:
  - Call energy splitting ΔE
  - Transitions between the two energy levels cause absorption or emission of electromagnetic (em) radiation for which $\Delta E = h\nu$
  - Resonance occurs when em radiation of frequency ν is injected

Magnetic resonance imaging

Magnetic resonance imaging (MRI) exploits this resonance

Steps:

- Apply uniform magnetic field, align proton (^{1}H) spins
- Apply radiation, at exactly ν, cause transitions between "spin up" & "spin down" states
- Turn off the radiation ... and ...
- "Listen" for radiation at exactly ν as the spins realign

Brilliant!
Simple principle and elegant technique, exploited in exquisitely sophisticated imaging systems.

Theoretical description; a hybrid of quantum and classical

Nuclear magnetic resonance & MRI are both inherently quantum mechanical effects:

- Signal is generated by manipulating the *spins* of hydrogen nuclei:
  - Spin is postulated to explain hyperfine structure, Stern-Gerlach experiment, ...
  - Understood theoretically through the symmetries of space and time
- Magnetic moment of proton, $\boldsymbol{\mu}$, is related to the proton spin, $\mathbf{s}$, by:

$$\boldsymbol{\mu} = \gamma \mathbf{s}$$

  where γ is the "gyromagnetic ratio"

Hybrid, quantum/classical treatment:

- Quantum mechanics: energy splitting and population in ground and excited state
- Classical: magnetisation vector, its precession, and the manipulation of the magnetisation vector to understand the signals used for imaging

Interaction of nuclear magnetic dipole with uniform magnetic field

The contribution, $\delta\mathcal{U}$, to the potential energy of a proton immersed in a magnetic field, **B**, is given by:

$$\delta\mathcal{U} = -\mathbf{B} \cdot \boldsymbol{\mu}$$

Lets consider a proton which, in the absence of a magnetic field, has energy E. Applying the magnetic field introduces $\delta\mathcal{U}$ into the Schrödinger equation resulting in a splitting of the proton energy level such that $E \to E'$ given by:

$$E' = E \pm E_{m_s}$$

where

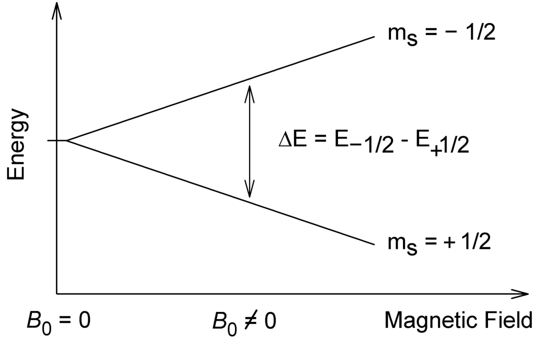
$$E_{m_s} = -m_s \gamma \hbar B_0$$

where m_s is the quantum number associated with the component of the proton spin parallel to **B**, $\hbar$ is Planck's constant divided by 2π, and B_0 is the magnitude of **B**

For the proton:

$$m_s = \pm\frac{1}{2}$$

Larmor equation



ΔE, splitting between two levels with $m_s = \pm\frac{1}{2}$:

$$\Delta E = \gamma\hbar B_0$$

Planck's law relates energy splitting to the angular frequency, ω, of the radiation required to excite the transition, therefore:

$$\Delta E = \hbar\omega$$

Writing ω in terms of γ and B_0 yields the Larmor equation:

$$\omega = \gamma B_0$$

Gyromagnetic ratios of some nuclei

Definition of gyromagnetic ratio, γ:

> *The gyromagnetic ratio, γ, of a particle or system is the ratio of its magnetic dipole moment to its angular momentum*

For charged body of charge q, mass m rotating about an axis of symmetry:

$$\gamma = \frac{qe}{2m}$$

where e is the magnitude of the charge on the electron

For proton, $q = 1$, $m = m_p$, the proton mass.

$\gamma\!\!\!^{-}$ is sometimes used instead of γ:

$$\bar{\gamma} = \frac{\gamma}{2\pi}$$

nucleus	γ (rad MHz T^{-1})	$\bar{\gamma} = \gamma / 2\pi$
^{1}H	267.513	42.576
^{2}H	41.065	6.536
^{3}He	203.789	32.434
^{7}Li	103.962	16.546
^{13}C	67.262	10.705
^{14}N	19.331	3.077
^{15}N	27.116	-4.316
^{17}O	36.264	5.772
^{19}F	251.662	40.053
^{23}Na	70.761	11.262
^{27}Al	69.763	11.103
^{31}P	108.291	17.235
^{57}Fe	8.681	1.382
^{63}Cu	71.118	11.319
^{67}Zn	16.767	2.669
^{129}Xe	73.997	11.777

Examples

Larmor equation: $\omega = \gamma B_0 \quad \Rightarrow \quad \nu = \bar{\gamma} B_0$

For hydrogen nucleus, ^{1}H, $\bar{\gamma} = 42.58\,\text{MHz/T}$

What is the resonance frequency for ^{1}H when:

- $B_0 = 1.5\,\text{T}$?
- $B_0 = 3.0\,\text{T}$?

What are the corresponding values for the energy splittings $\Delta E = h\nu$, where h is Planck's constant?

Summary of section 1

Magnetic moment of proton exploited to provide energy splitting, ΔE, between spin-up and spin-down states in applied magnetic field

Injection of radio-frequency wave with a frequency that resonates with the splitting then used to manipulate population of protons in the spin-up and spin down states

Images produced by manipulating applied magnetic field and frequency of RF field gradients

Larmor frequency, ω determined by the gyromagnetic ratio, γ, and the applied magnetic field, B_0: $\omega = \gamma B_0$