CCAP
Centre for the Clinical Application of Particles

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Physics of Medical Imaging and Radiotherapy

Magnetic Resonance Imaging

Lecture 1; Section 3: Classical derivation of the Larmor equation

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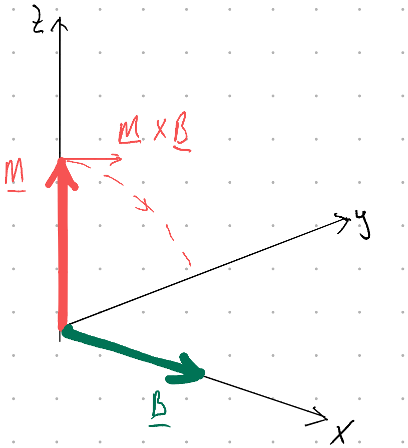
Section 1

Rotating the magnetisation

First, a static example



Consider magnetisation **M** parallel to z axis and **B** parallel to the x axis, as shown

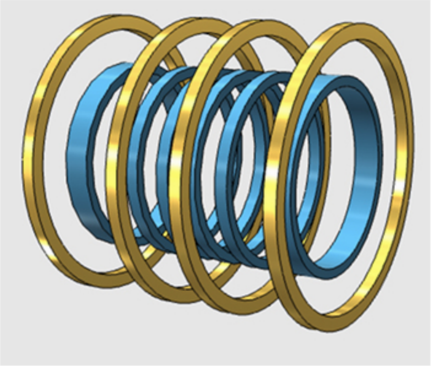
Torque, $\mathbf{M} \times \mathbf{B}$, is therefore parallel to the y axis

Net result is that **M** will precess around the x axis towards the y axis

This is what is done in MRI . . .

Rotating the magnetisation vector in MRI; principle

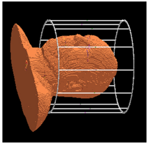
Main field, $\mathbf{B}_0$, produced with solenoid



Induces magnetisation $\mathbf{M}$ parallel to $\mathbf{B}_0$

To rotate $\mathbf{M}$ away from $\mathbf{B}_0$ require magnetic field in transverse (x, y) plane

Call the field in the x, y plane $\mathbf{B}_1$; can be produced with a variety of coil arrangements, e.g. dipole or, more efficient, a "bird cage"



To cause $\mathbf{M}$ to precess require that magnetic field oscillates at the Larmor frequency, ω.

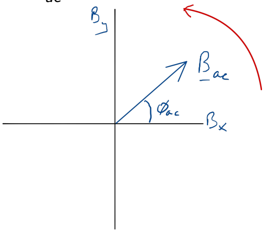
…i.e. require RF magnetic field $\mathbf{B}_1$

Rotating the magnetisation vector in MRI; mathematics

Take $\mathbf{B}_1$ to be "plane polarised" in x, y such that $B_{1_x} = B_1 \cos(\omega t + \alpha)$ and $B_{1_y} = B_1 \sin(\omega t + \beta)$; α and β are phases

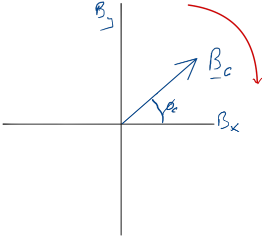
$\mathbf{B}_1$ can be rewritten in terms of two circularly polarised fields:

$\mathbf{B}_{1_{ac}}$; anti-clockwise



$$B_{1_{ac}} = \frac{B_1}{2};\ \phi_{ac} = \omega t + \alpha'$$

$\mathbf{B}_{1_c}$; clockwise



$$B_{1_c} = \frac{B_1}{2};\ \phi_c = \omega t + \beta'$$

Rotating the magnetisation vector in MRI

One of the two counter rotating fields will rotate in the same direction as the nuclear precession

Either $B_{1_{ac}}$ or B_{1_c} will appear stationary in the plane transverse to $\mathbf{B}_0$ in the frame that is co-rotating with the precession of the net magnetisation vector. Call the co-rotating field B_1^+

B_1^+ is equal to either $B_{1_{ac}}$ or B_{1_c} depending on the direction of $\mathbf{B}_0$

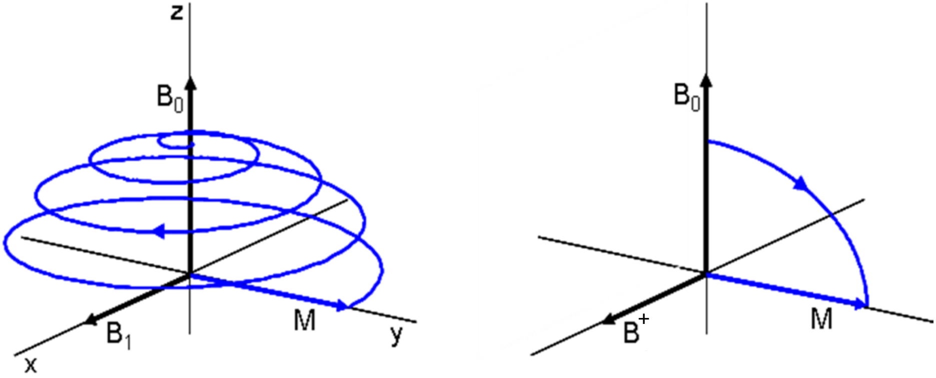
The field stationary in the rotating frame will therefore cause **M** to precess about a rotating axis in the transverse plane

The net result is that **M** can be rotated into the x, y plane where it will continue to precess

The precession of **M** in the x, y plane gives a detectable RF signal

Rotating the magnetisation vector in MRI

M is initially parallel to $\mathbf{B}_0$



(a) Laboratory Frame of Reference

(b) Rotating Frame of Reference

The flip angle

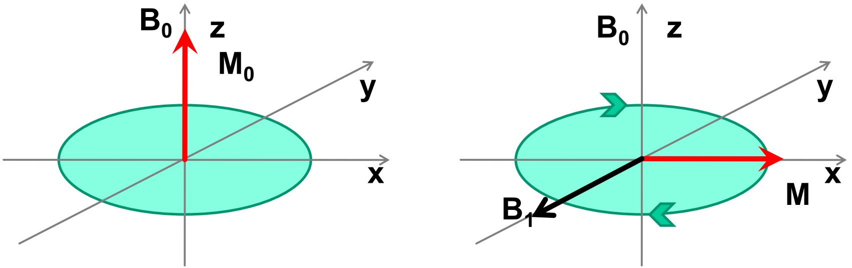
The flip angle, α, is proportional to the magnitude and duration of the RF pulse:

$$\alpha = \gamma B_1 t_P$$

where t_P is the duration of the RF pulse

90° pulse rotates magnetisation into transverse plane where it continues to precess

Effect of 90° RF Pulse



Example: calculating the duration of a 90° pulse

RF transverse magnetic field pulse is applied to rotate $\mathbf{M}$

The magnitude of B_1 is $10\,\mu$T (i.e. 10^{-5} T)

At what rate will $\mathbf{M}$ rotate away from the $\mathbf{B}_0$ axis?

How long will it take for the flip angle to reach 90°?

Example: calculating the duration of a 90° pulse

Half an answer . . .

The magnitude of B_1 is $10\,\mu$T (i.e. 10^{-5} T)

At what rate will $\mathbf{M}$ rotate away from the $\mathbf{B}_0$ axis?

It will rotate at the Larmor frequency, f_1, arising from the field B_1, i.e. $f_1 = \bar{\gamma} B_1$

How long will it take for the flip angle to reach 90°?

The angle can be obtained by solving the equation:

$$\frac{1}{4} = \bar{\gamma} B_1 t_P^{90^\circ} \text{ for } t_P^{90^\circ}$$

$$\text{or } \frac{\pi}{2} = \gamma B_1 t_P^{90^\circ}$$

Summary of section 3

Net magnetisation of ^{1}H spins caused to rotate using plane-polarised, time-varying magnetic field in the x, y plane

Precession of rotated net-magnetisation vector gives rise to RF signal which can be detected

Measurement of the RF signal from the precession of rotated net-magnetisation vector is the basis of MRI