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Physics of Medical Imaging and Radiotherapy

Magnetic Resonance Imaging

Lecture 2; Relaxation of the magnetisation and spatial encoding

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Contents

Section 1

Determination of the spin-lattice relaxation time, T_1

What does it take to make an MRI image

NMR can be used to generate signals that depend on the concentration of ^{1}H in tissue; the basis of an imaging technique

The spin-lattice and spin-spin relaxation times, T_1 and T_2 respectively, depend on tissue type—so can be used to distinguish neighbouring tissues

To generate an image need to:

- Extract T_1 and T_2; and
- Spatially localise the signal

Relaxation times revisited

Tissue Type	T1 (ms)	T2 (ms)
Adipose tissues	240-250	60-80
Whole blood (deoxygenated)	1350	50
Whole blood (oxygenated)	1350	200
Cerebrospinal fluid (similar to pure water)	4200 - 4500	2100-2300
Gray matter of cerebrum	920	100
White matter of cerebrum	780	90
Liver	490	40
Kidneys	650	60-75
Muscles	860-900	50

Relaxation times characteristic of tissue type

For materials important for human imaging
$T_1 > T_2$

T_1 characteristic of recovery of longitudinal magnetisation

T_2 must be extracted from the decay of the transverse magnetisation which is characterised by T_2^* which is related to T_2 by:

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'}$$

Free induction decay revisitted

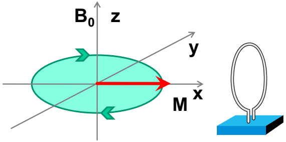
System set up in equilibrium; net magnetisation, $\mathbf{M}_0$, parallel to $\mathbf{B}_0$ and of magnitude M_0

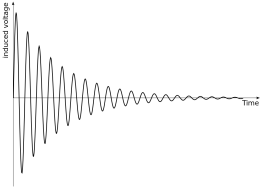
90° RF magnetic field pulse applied to rotate net magnetisation, $\mathbf{M}_0$, into x, y plane

Take $t = 0$ to be time at which 90° degree pulse ends. Magnitude of transverse magnetisation, M_{xy}, at $t = 0$:

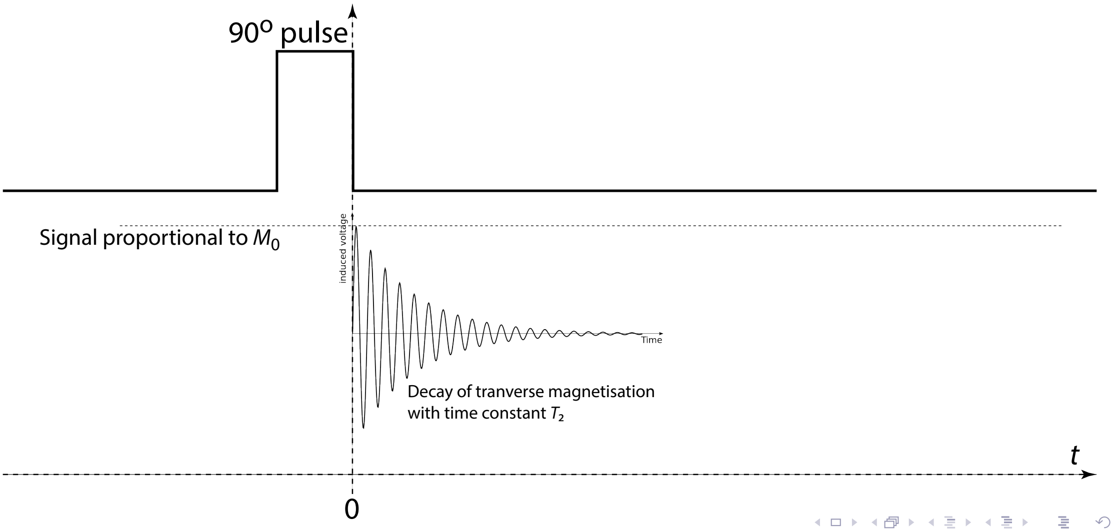
$$M_{xy}(t = 0) = M_{xy}(0) = M_0$$

M_{xy} decays exponentially, as described in lecture 8





Application of 90° pulse and spin-spin relaxation time, T_2



Recovery of longitudinal polarisation and spin-lattice relaxation time, T_1

Longitudinal magnetisation recovers according to:

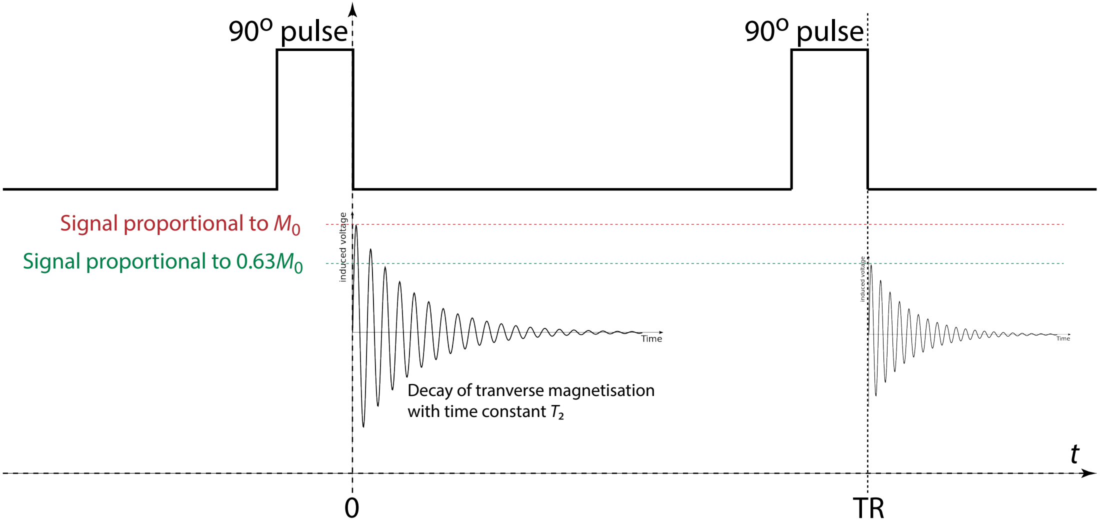
$$M_z(t) = M_0 \left[1 - \exp\left(-\frac{t}{T_1} \right) \right]$$

So, for $t \gtrsim 5T_1$, $M_z - M_0 \lesssim 0.5\%$, i.e. longitudinal magnetisation has recovered

If a second 90° pulse is applied for $t < 5T_1$ then the resulting M_{xy} will be less than M_0

For example, if the second 90° pulse is applied at $t = T_1$, then $M_{xy}(t = T_1) = 0.63M_0$

Applicaton of multiple 90° pulses



The time to repetition, TR, and extraction of T_1

Longitudinal magnetisation recovers according to:

$$M_z(t) = M_0 \left[1 - \exp\left(-\frac{t}{T_1}\right)\right]$$

So, for $t \gtrsim 5T_1$, $M_z - M_0 \lesssim 0.3\%$, i.e. longitudinal magnetisation has recovered

If a second 90° pulse is applied for $t < 5T_1$ then the resulting M_{xy} will be less than M_0

In general:

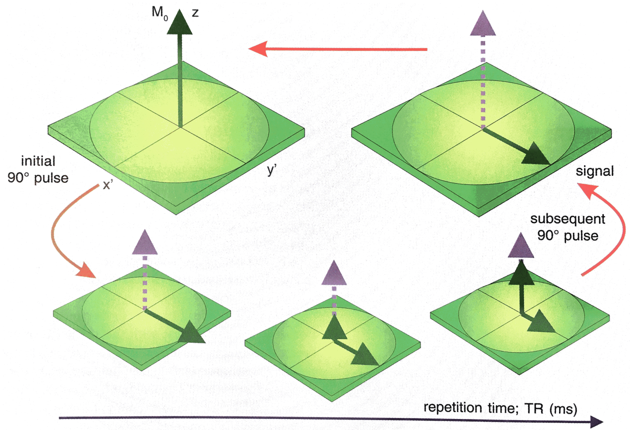
$$M_z(\mathrm{TR}) = M_0 \left[1 - \exp\left(-\frac{\mathrm{TR}}{T_1}\right)\right]$$

So, repetition of 90° pulse at $t = \mathrm{TR}$ gives $M_{xy}(\mathrm{TR}) = M_z(\mathrm{TR})$

Can extract T_1 by measuring $M_{xy}(\mathrm{TR})$ as a function of TR

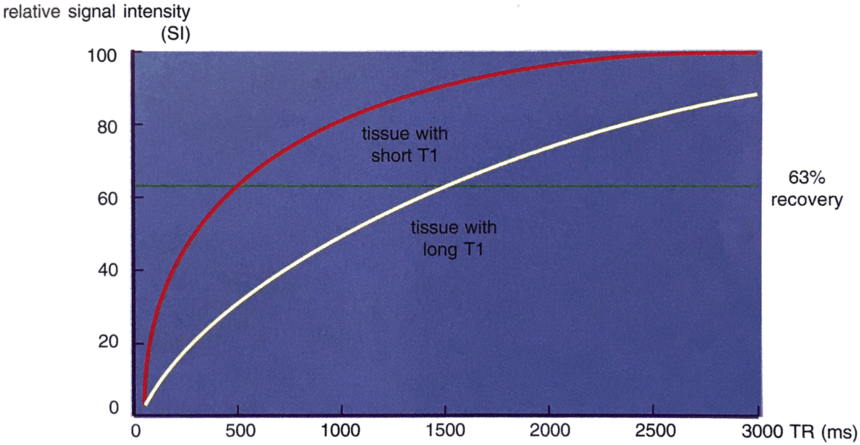
Partial saturation pulse sequence

"**Partial saturation pulse sequence**", graphical representation of evolution of magnetisation

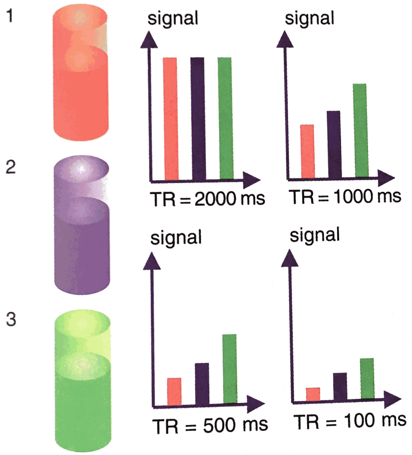


Extraction of spin-lattice relaxation time constant, T_1

Comparison of relative signal intensity in partial saturation sequence for two different tissues



Using TR to distinguish between different tissues



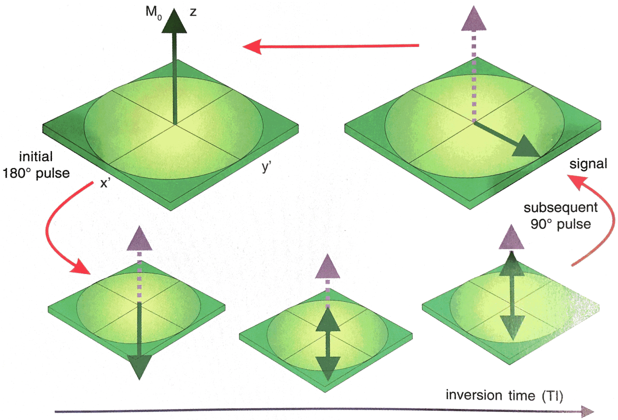
Example three types of tissue:

1. Blood: $T_1 = 1350\,\text{ms}$
2. Muscle: $T_1 = 875\,\text{ms}$
3. Fat: $T_1 = 230\,\text{ms}$

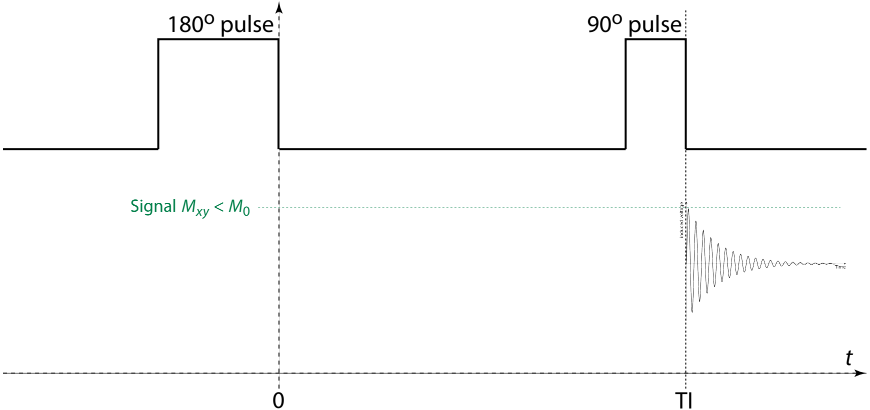
Note how tissues can be distinguished by comparing signal behaviour as a function of TR

Inversion recovery pulse sequence and time to inversion, TI

"**Inversion recovery pulse sequence**", graphical representation of evolution of magnetisation



Inversion recovery: using TI to extract T_1



$$M_z(\mathrm{TI}) = M_0 \left[1 - 2\exp\left(-\frac{\mathrm{TI}}{T_1}\right)\right]$$

Summary of section 1

T_1, the longitudinal or spin-lattice, relaxation time constant can be reconstructed using pulse sequences in which the net magnetisation is repeatedly rotated into the x, y plane and the evolution of the maximum of the transverse magnetisation is observed

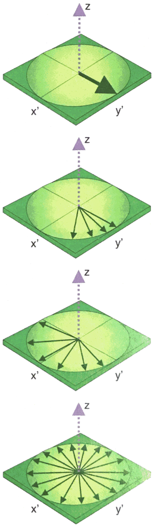
Pulse sequences used to obtain T_1:

- 90° pulse sequence
- Inversion recovery pulse sequence

Section 2

Determination of the spin-spin relaxation time, T_2

Spin-spin relaxation time, T_2



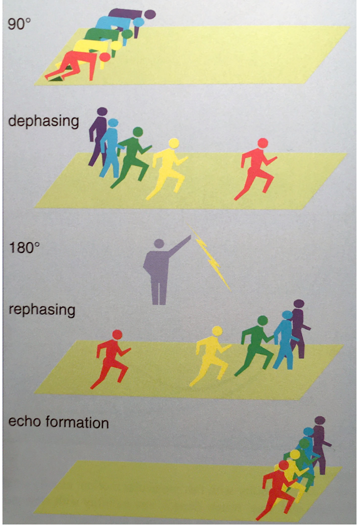
System set up in equilibrium; net magnetisation, $\mathbf{M}_0$, parallel to $\mathbf{B}_0$ and of magnitude M_0

90° RF magnetic field pulse applied to rotate net magnetisation, $\mathbf{M}_0$, into x, y plane

Take $t = 0$ to be time at which 90° degree pulse ends. At this instant net magnetisation begins to precess around $\mathbf{B}_0$

Rate of precession of individual ^{1}H nuclei depends on local magnetic environment: some precess faster, some slower. Results in decoherence, time constant T_2^* (see lecture 8)

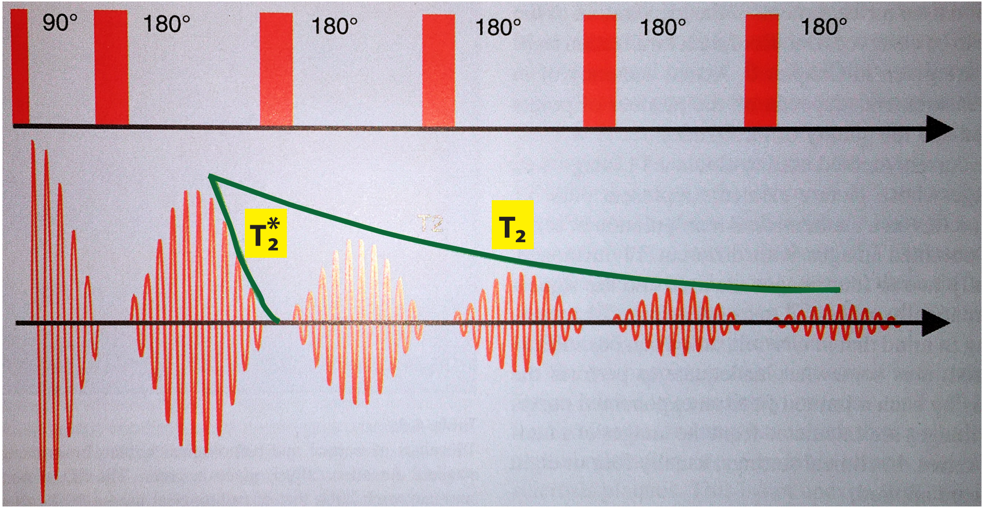
Spin-spin relaxation time, T_2



Before "doing the spins", an analogy:

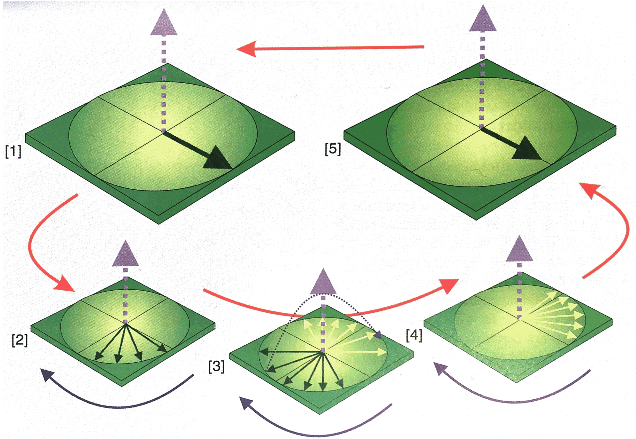
- A set of sprinters have been prepared at the starting line
- The "starting gun" is the end of the 90° pulse
- The sprinters run for a period of time, $t_{\rm run}$
- At $t_{\rm run}$ the sprinters' phase is rotated by 180°:
  The first becomes the last, etc.
- After a further $t_{\rm run}$ all sprinters are back in line
- The line of sprinters at $t = 2t_{\rm run}$ is an "echo" of the situation at $t = 0$

Spin-spin relaxation time, T_2



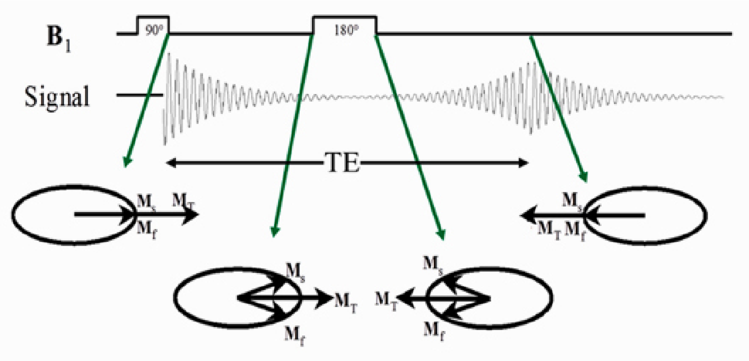
The spin-spin relaxation time constant, T_2

"**Spin echo sequence**", graphical representation of evolution of magnetisation

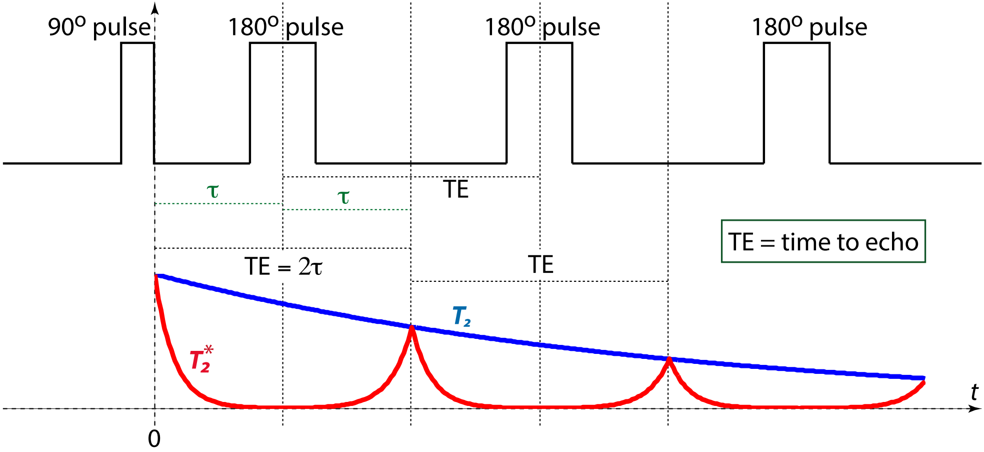


The spin-spin relaxation time constant, T_2

"**Spin echo sequence**", graphical representation of evolution of magnetisation



Spin-spin relaxation time, T_2



$$M_{xy}(\mathrm{TE}) = M_0 \exp\left(-\frac{\mathrm{TE}}{T_2}\right)$$

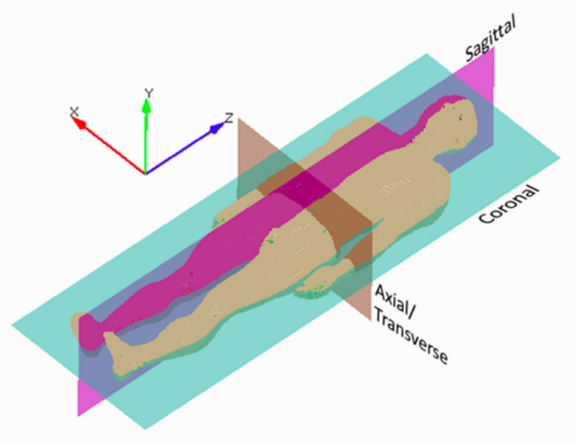
Summary of section 2

T_2, the spin-spin, relaxation time constant can be reconstructed using a spin-echo pulse sequence

Section 3

Slice selective excitation

Introduction



Conventional terminology & orientation of RH coordinate system

Contrast between tissues is afforded by RF B_1 pulse sequences such as those discussed in the preceding lectures

To make an image, need to localise the signals to appropriately small regions of space

To localise signals exploit:

- Resonance, i.e. Larmor frequency $\nu = \gamma B$
- By making B a function of position

i.e. make ν a function of position:

$$\nu(x, y, z) = \gamma B(x, y, z)$$

Slice selective excitation

Goal: excite a slice of tissue of thickness δ

So far a uniform "main field" $\mathbf{B}_0 = B_0\hat{\mathbf{k}}$ has been considered

Require to make B_z a function of position to make Larmor frequency position dependent

Apply "gradient" fields G_i such that B_z becomes:

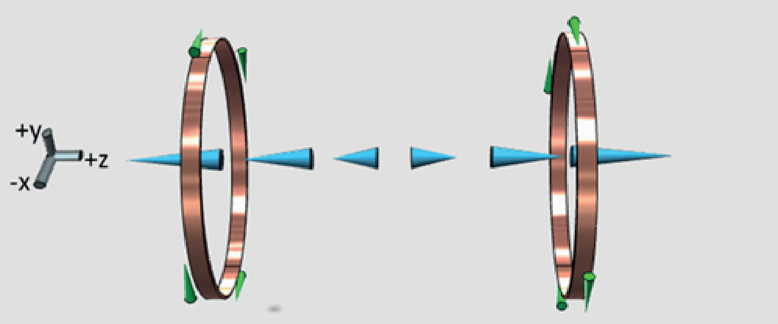
$$B_z(x, y, z, t) = B_0 + xG_x(t) + yG_y(t) + zG_z(t)$$

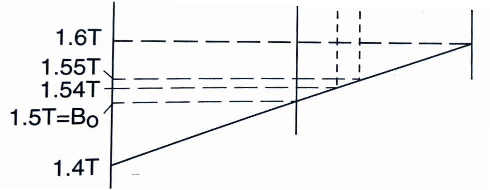
Ideally G_i only have one field component directed along the z direction so that:

$$\mathbf{B} = B_z(x, y, z, t)\hat{\mathbf{k}}$$

With appropriate choice of G_i can generate a field gradient in any direction

Transverse slice; i.e. plane at fixed z





Example:
Helmholtz coils in opposition

Ideal gradient:
$G_z = \text{constant}$

Transverse slice; slice thickness and bandwidth

Lets say that response needs to be isolated to a slice: $\delta z = 5\,\mathrm{mm}$ centred about $z = 0$

Take:

- The magnitude of the main field to be $B_0 = 1.5\,\mathrm{T}$
- The field gradient $G_z = 50\,\mathrm{mT\,m^{-1}}$
- $\gamma = 42.58\,\mathrm{MHz\,T^{-1}}$

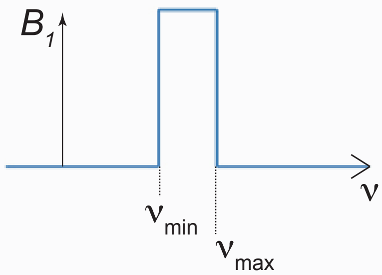
Take the slice to be $-2.5 < z < 2.5\,\mathrm{mm}$, then the Larmor frequency will run over the following range:

$$\begin{aligned} \nu_{\mathrm{min}} &= (1.5 - 0.125 \times 10^{-3}) \times 42.58 \approx 63.8646\,\mathrm{MHz} \\ \nu &= 1.5 \times 42.58 \approx 63.87\,\mathrm{MHz} \\ \nu_{\mathrm{max}} &= (1.5 + 0.125 \times 10^{-3}) \times 42.58 \approx 63.8753\,\mathrm{MHz} \end{aligned}$$

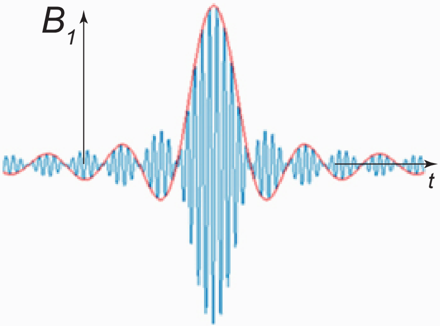
So, the spread of frequencies, the **bandwidth**, $\Delta\nu$ is:

$$\Delta\nu = 63.8646 - 63.8753 \approx 10.7\,\mathrm{kHz}$$

Transverse slice; excitation of spins in slice



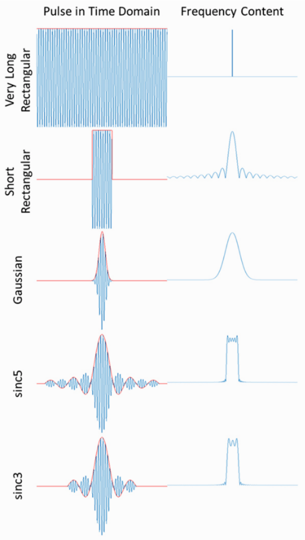
Idealised, square frequency distribution



Fourier transform of square frequency distribution

B_1 oscillates at ν, amplitude is modulated according to "sinc" function (red line)

Transverse slice: excitation pulses



Frequency content of a variety of excitation pulses:

- *Very long rectangular*: narrow band of Larmor frequencies
- *Short rectangular*: frequency distribution follows "sinc" function:

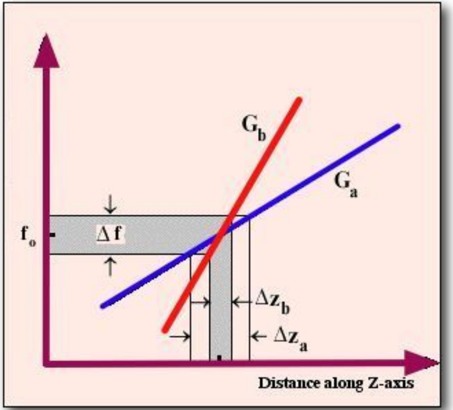
$$A(\nu) \propto \mathrm{sinc}(\nu) = \frac{\sin \nu}{\nu}$$

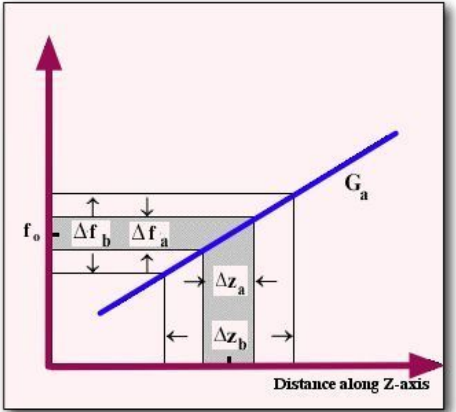
  where $A(\nu)$ is the amplitude of contribution at frequency ν
- *Gaussian*: Fourier transform of Gaussian in t is a Gaussian in ν
- *sincN*: Since square pulse requires contributions over all ν, the frequency range is often truncated. The "sincN" function represents a sinc function for which the frequency range is truncated after N zero crossings

Transverse slice: determining the slice thickness

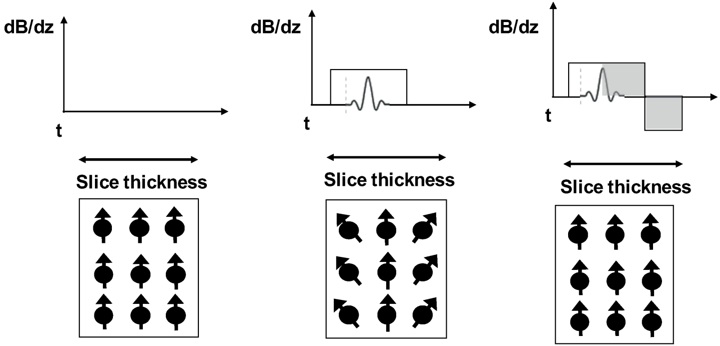
Slice thickness is determined by bandwidth ($\Delta\nu$) and field gradient (G_z)

Sorry for the change in notation!





Transverse slice: spin rephasing pulse



Larmor frequency across slice changes. So, over the time that the gradient pulse is applied, the spins precess at different rates

Therefore, at the end of the pulse the phase of the spins differs as a function of z

A rephasing pulse which reverses the field gradient (i.e. for which $G_z \rightarrow -G_z$) is applied

Transverse slice: spin rephasing pulse

Size of the spin rephasing pulse is determined by considering the rate at which the phase difference accumulates

Rate of precession is given by the Larmor frequency, ω, so change in phase of a spin during the gradient pulse is given by:

$$\Phi = \omega\tau = \gamma\left(B_0 + zG_z\right)\tau$$

where τ is the length of the gradient pulse in time

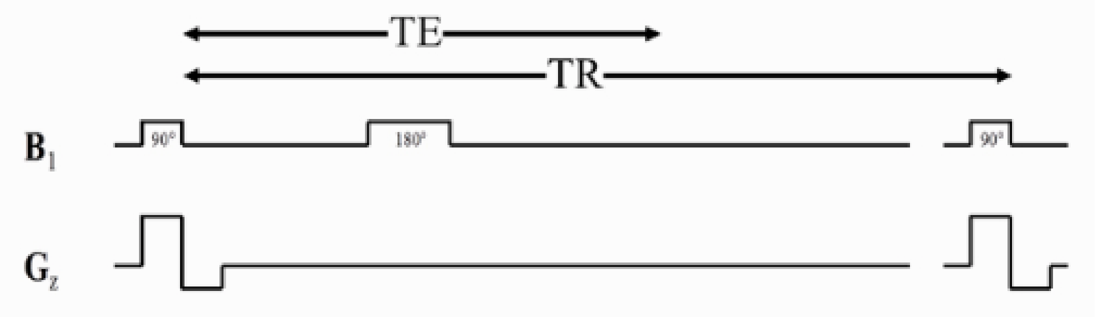
So, phase difference between edges of the slice and the centre is:

$$\Delta\Phi = \gamma\tau G_z \frac{\delta z}{2}$$

So, rephasing pulse, G_z^{rephase}, and the length over which it is applied, τ^{rephase} must satisfy:

$$G_z^{\mathrm{rephase}}\tau^{\mathrm{rephase}} = G_z\frac{\tau}{2}$$

Transverse slice: partial spin-echo pulse sequence



B_1 rotates net magnetisation in the selected slice with gradient pulse applied

Summary of section 3

Localisation of MRI signal to plane achieved using magnetic-field gradient combined with frequency-dependent readout of radiowave generated by precession of net magnetisation

Frequency content of RF (B_1) pulse determines spread of perturbations to Larmor frequency

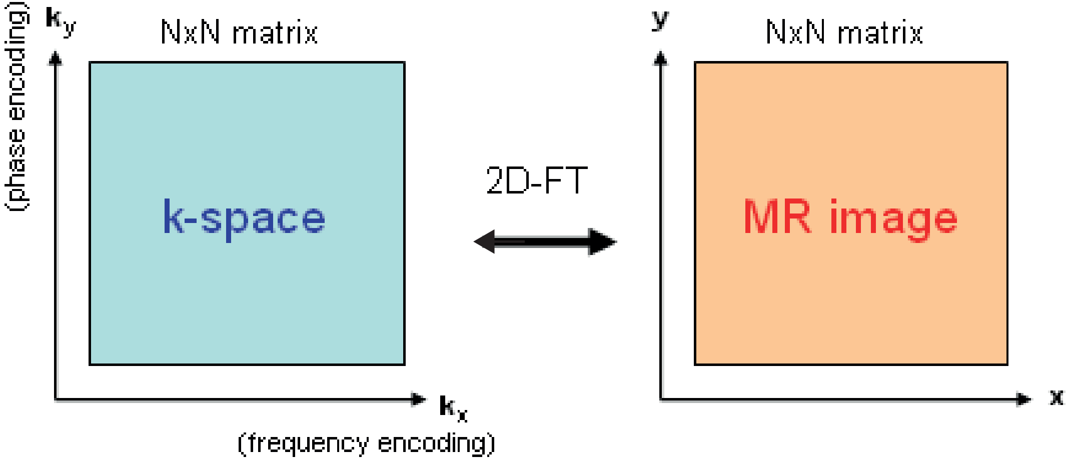
Spin-rephasing pulse applied after main B_1 pulse. Length of rephasing pulse required is half that of the main B_1 pulse

Section 4

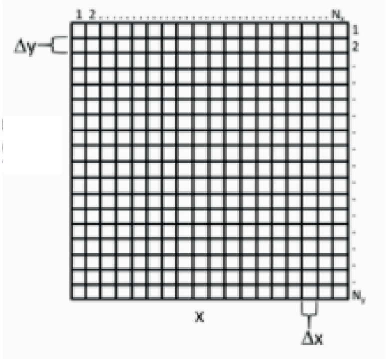
Encoding spatial information in k-space

Encoding spatial information into the net magnetisation

The basis is a 2D Fourier transform:



2D Fourier transformation



2D image in "coordinate space", x, y, presented in pixel grid

Field of view, FOV, in coordinate space:

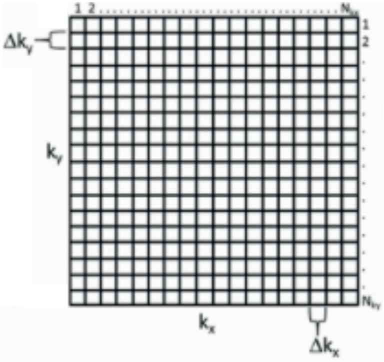
$$(x_{\mathrm{max}} - x_{\mathrm{min}}, y_{\mathrm{max}} - y_{\mathrm{min}})$$

Pixel size (resolution):

$$\Delta x = \frac{x_{\mathrm{max}} - x_{\mathrm{min}}}{N_x}$$

$$\Delta y = \frac{y_{\mathrm{max}} - y_{\mathrm{min}}}{N_y}$$

2D Fourier transformation



2D image in "k space", k_x, k_y, presented in pixel grid

Field of view, FOV, in k space:

$$(k_{x\,\mathrm{max}} - k_{x\,\mathrm{min}}, k_{y\,\mathrm{max}} - k_{y\,\mathrm{min}})$$

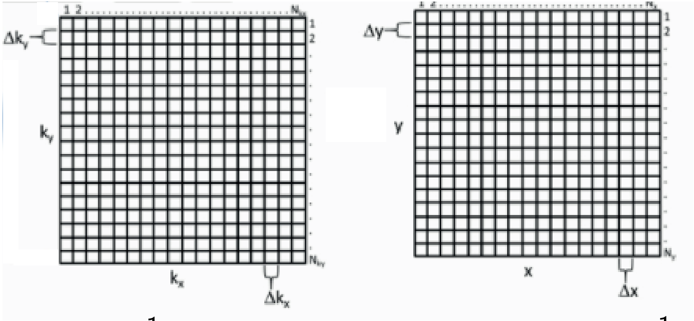
Pixel size (resolution):

$$\Delta k_x = \frac{k_{x\,\mathrm{max}} - k_{x\,\mathrm{min}}}{N_x}$$

$$\Delta k_y = \frac{k_{y\,\mathrm{max}} - k_{y\,\mathrm{min}}}{N_y}$$

2D Fourier transformation

Transformation between resolution in coordinate-space and k-space representations:



$$\Delta k_x = \frac{1}{(x_{\rm max} - x_{\rm min})} \qquad \Delta x = \frac{1}{(k_{x\,\rm max} - k_{x\,\rm min})}$$

$$\Delta k_y = \frac{1}{(y_{\rm max} - y_{\rm min})} \qquad \Delta y = \frac{1}{(k_{y\,\rm max} - k_{y\,\rm min})}$$

2D Fourier transformation

Define $\rho(x, y)$ to be the intensity pixel-by-pixel in coordinate space.

2D Fourier transform from coordinate to k space is then:

$$S\left(k_x, k_y\right) = \int_{y_{\min}}^{y_{\max}} \int_{x_{\min}}^{x_{\max}} \rho\left(x, y\right) \exp\left(-i2\pi k_x x\right) \exp\left(-i2\pi k_y y\right) dxdy$$

where $S(k_x, k_y)$ is the intensity pixel-by-pixel in k space

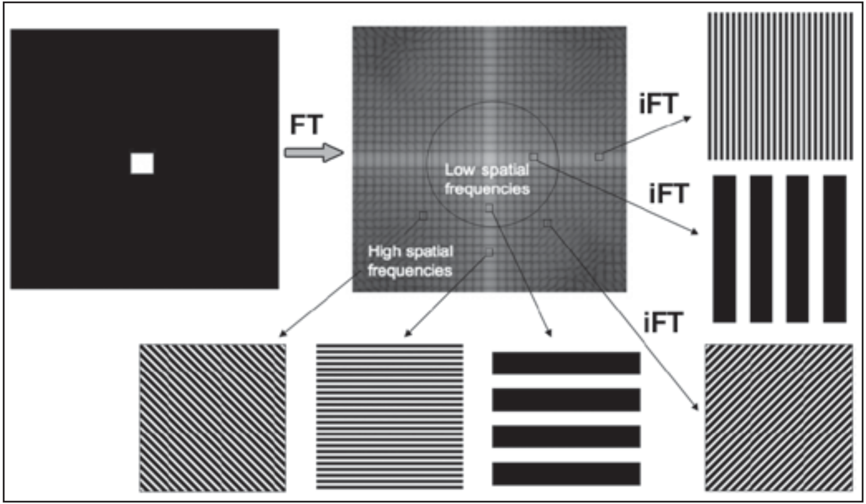
Inverse Fourier transform takes k-space intensity map to coordinate-space intensity map:

$$\rho\left(x, y\right) = \int_{k_{y\,\min}}^{k_{y\,\max}} \int_{k_{x\,\min}}^{k_{x\,\max}} S\left(k_x, k_y\right) \exp\left(i2\pi k_x x\right) \exp\left(i2\pi k_y y\right) dk_x dk_y$$

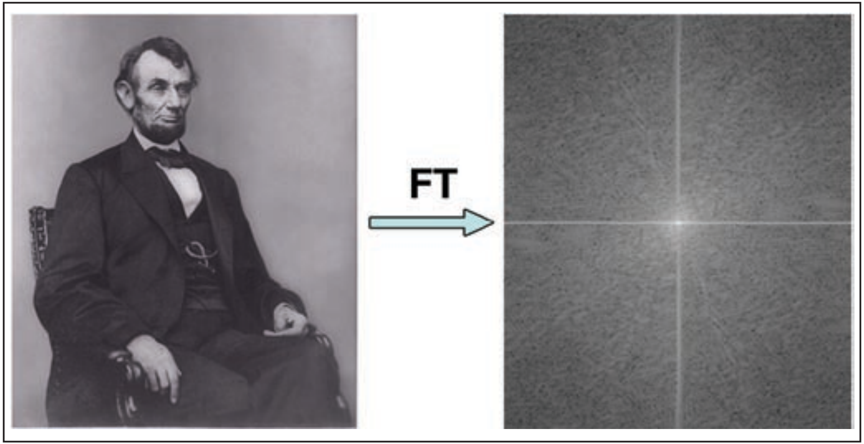
Example one: a single dot

Example two: three dots

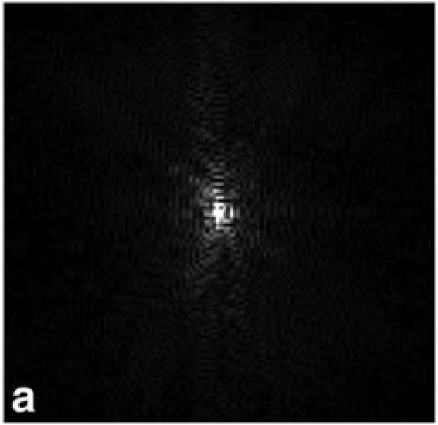
Example three: Square in centre of field of view



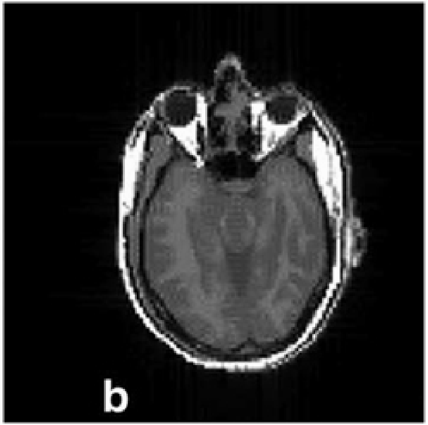
Example four: Abraham Lincoln



Example five: Section through skull



(a) k-space image of head



(b) coordinate-space image of head

Challenge: record k-space image using NMR signals

Summary of section 4

Intensity distribution in "coordinate space" ($\rho(x, y)$) mapped using a Fourier transform onto intensity distribution in "k"-space ($S(k_x, k_y)$)

Signals generated in MRI scan recorded in k-space; coordinate space image obtained by inverse Fourier transform