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Magnetic Resonance Imaging

Lecture 3; Section 2: MRI image reconstruction; reprise

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Section 2

Image reconstruction in MRI; reprise

Artefacts in MRI

Ghosting due to total internal reflection of bright sources in optical photography

Just as in optical photography, artefacts are unwanted image features

Artefacts arise from many causes:

- Field imperfections (not addressed below)
- Movement of patient or organ
- Magnetic material (e.g. from bone repairs)
- Chemical composition uncertainties

My objective is to give examples, there is an extensive literature on the subject

Reconstruction of the MR image; reprise

Gradient pulses G_i are used to allow slice-selective excitation and to allow spatial information to be encoded into the net magnetisation:

$$B_z(x, y, z, t) = B_0 + xG_x(t) + yG_y(t) + zG_z(t)$$

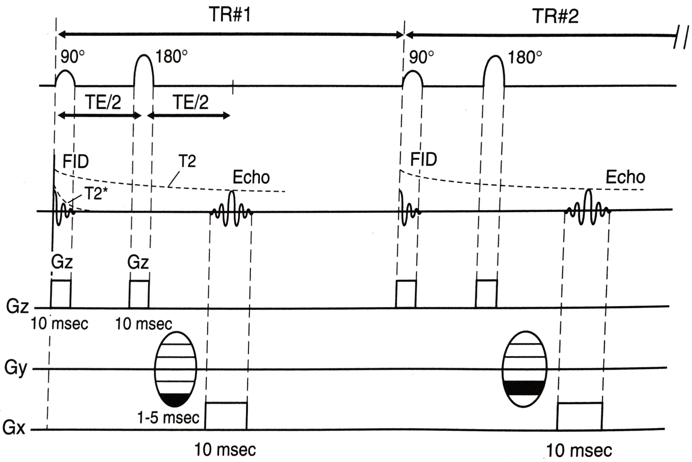
Spatial information is encoded into net magnetisation in k-space, often:

- Frequency encoding is used to encode features in the x direction
- Phase encoding is used to encode features in the y direction

2D Fourier transform used to transform image in k space to image in coordinate space

Pulse sequence is repeated to collect data for all $N_x \times N_y$ pixels of image

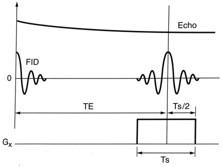
Spin-echo sequence; reprise



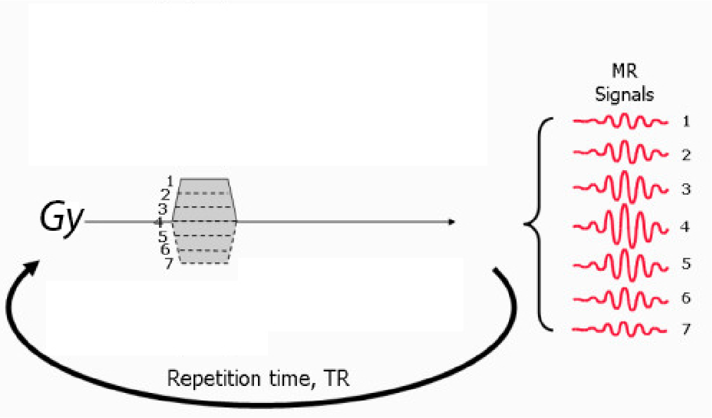
Readout occurs when frequency-encoding pulse (G_x) is on (T_S, the sampling time)

Each repetition corresponds to a new G_y, i.e. a new encoding of phase

Take N_y repetitions to fill N_y rows in the image



Phase encoding; reprise (1 of 3)

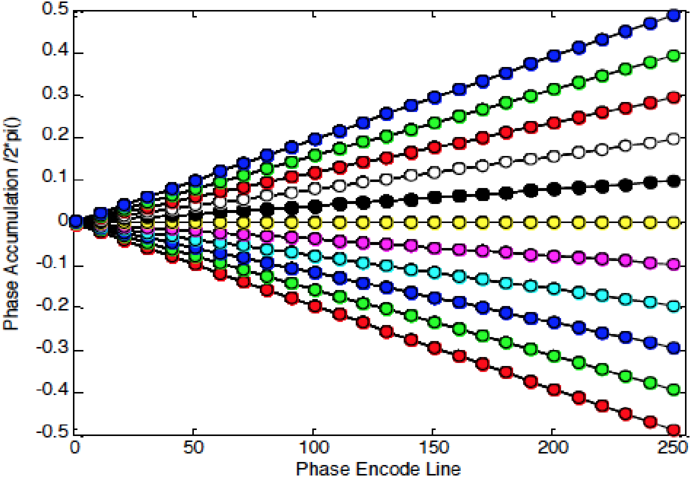


Central line in k-space will contain phase-encoding step with weakest gradient and strongest signal

Periphery of k-space sill contain phase-encoding steps with the strongest gradients and weakest signal

Each slice "has its own k-space because excitation is tuned to G_z

Phase encoding; reprise (2 of 3)



Phase wrt $y = 0$ accumulates with time, t, while phase-encoding gradient, G_y, is on:

$$\Phi(G_y, y, t) = (\gamma G_y)\, yt \qquad (1)$$

The slope of the line in the figure is determined by y

I.e. the rate of change of phase (frequency) is given by:

$$\frac{\Delta\Phi}{\Delta t} = (\gamma G_y)\, y$$

2D Fourier transform revisited

If the phase-encoding pulse is of length τ_{pe}, then the change of phase of the spins relative to $y = 0$ at the end of the pulse will be given by:

$$\frac{\Delta\Phi}{\Delta y} = (\gamma G_y \tau_{pe})$$

Lets take the start of the frequency-encoding pulse, G_x, to be at $t = 0$, then, the phase advance of ^{1}H nuclei at x after time t will be:

$$\phi(G_x, x, t) = (\gamma G_x)xt \qquad \text{i.e.} \qquad \frac{\Delta\phi}{\Delta x} = (\gamma G_x)t$$

As in lecture 10, let $\rho(x, y)$ be the intensity pixel-by-pixel in coordinate space, then the signal S will be given by:

$$S\left(G_y, \tau_{pe}, G_x, t\right) = \int_{y_{\rm min}}^{y_{\rm max}} \int_{x_{\rm min}}^{x_{\rm max}} \rho\left(x, y\right) \exp\left[-i\left(\gamma G_x t\right) x\right] \exp\left[-i\left(\gamma G_y \tau_{pe}\right) y\right] dxdy$$

2D Fourier transform revisited

In lecture 10, 2D Fourier transform from coordinate to k space was given as:

$$S\left(k_x, k_y\right) = \int_{y_{\rm min}}^{y_{\rm max}} \int_{x_{\rm min}}^{x_{\rm max}} \rho\left(x, y\right) \exp\left(-i2\pi k_x x\right) \exp\left(-i2\pi k_y y\right) dxdy$$

where $S(k_x, k_y)$ is the intensity pixel-by-pixel in k space

If we identify:

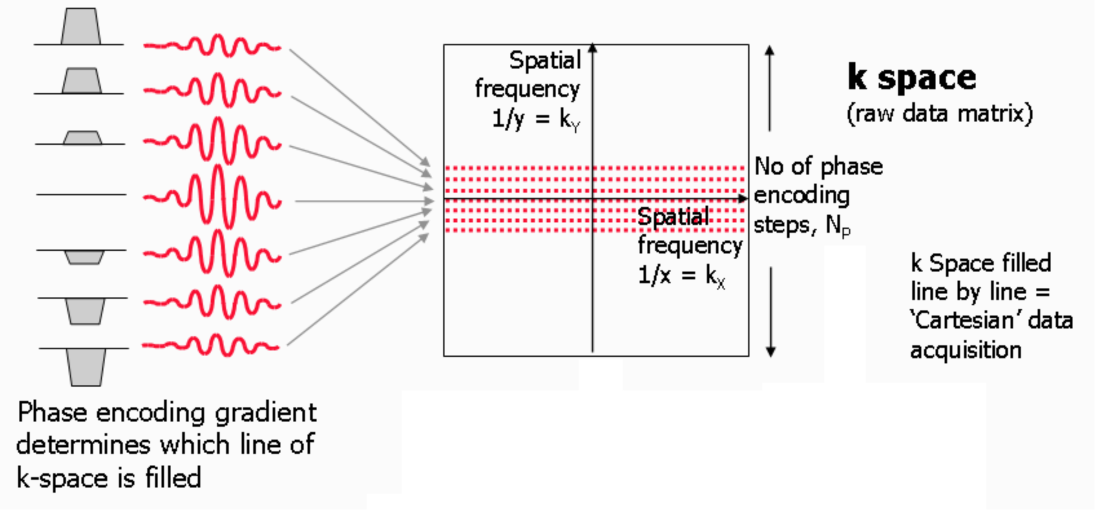
$$\begin{aligned} k_x &= \frac{\gamma}{2\pi} G_x t \\ k_y &= \frac{\gamma}{2\pi} G_y \tau_{pe} \end{aligned}$$

then:

$$S\left(k_x, k_y\right) = S\left(G_y, \tau_{pe}, G_x, t\right)$$

And the measured signal, S, is the k-space representation of the coordinate-space intensity ρ

Phase encoding; reprise (3 of 3)



Summary of section 2

The slice is selected by tuning the RF frequency and G_z

The k_x coordinate is obtained from frequency encoding **at readout**

The k_y coordinate is obtained from phase encoding **“passively” by manipulating phase during free induction decay (FID)**