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Physics of Medical Imaging and Radiotherapy

Magnetic Resonance Imaging

Lecture 3; Image reconstruction and introduction to artefacts in MRI

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Contents

Section 1

Encoding spatial information into net magnetisation

Spatial encoding and field gradients

Gradient pulse causes Larmor frequency to become a function of position

So, the phase of the nuclear precession will become a function of position over the period of a gradient pulse

Exploit these features to:

- Encode x position into k_x via "frequency encoding"
- Encode y position into k_y via "phase encoding"

Remember, gradient pulses G_i are such that:

$$B_z(x, y, z, t) = B_0 + xG_x(t) + yG_y(t) + zG_z(t)$$

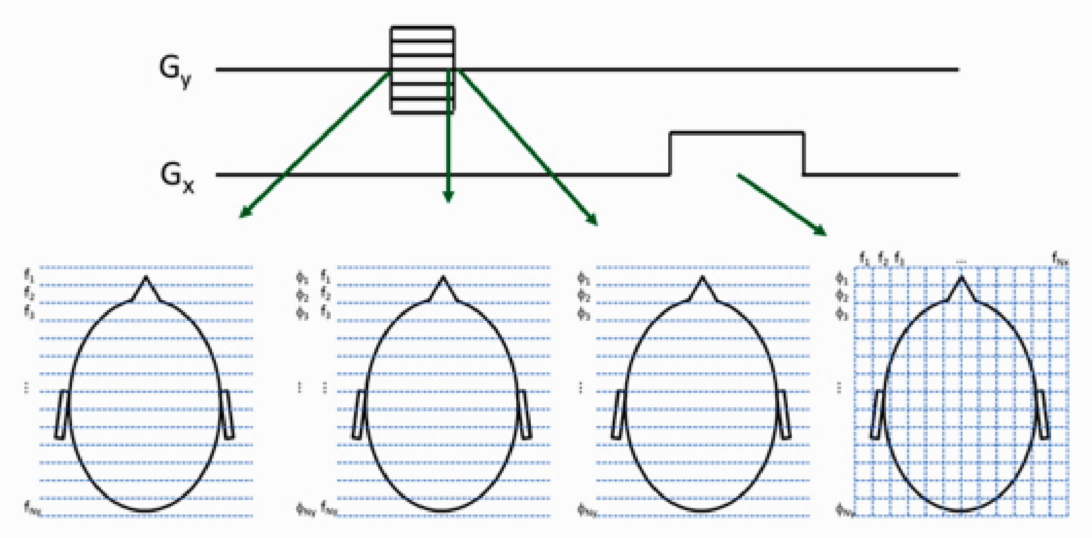
$G_x = \frac{\partial B_z}{\partial x}$; i.e. a magnetic-field gradient in x direction

magnetic field xG_x is in the $\hat{\mathbf{k}}$ direction

$G_y = \frac{\partial B_z}{\partial y}$; i.e. a magnetic-field gradient in y direction

magnetic field yG_y is in the $\hat{\mathbf{k}}$ direction

Conversion of field gradient into *k* space

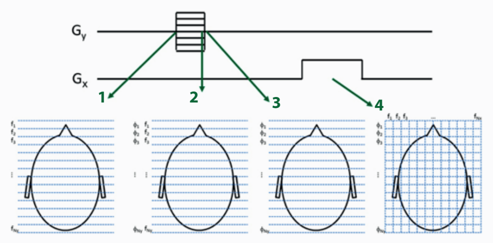


Conversion of field gradient into k space

Example:

phase encode y,

frequency encode x

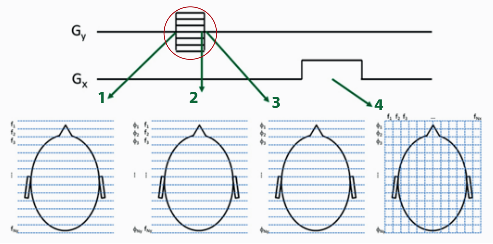


1. At start of phase encoding-pulse, spins are in phase. G_y causes Larmor frequency to be function of y: $\nu = f(y)$
2. At end of phase encoding pulse, phase of precession, ϕ, has become a function of y, i.e. $\phi \to \phi(y)$
3. As time passes, phase dependence on y is preserved, i.e. $\phi = \phi(y)$
4. Gradient pulse G_x causes Larmor frequency to become a function of x. Result is that y-position information is encoded in $\phi = \phi(y)$ and x-position information is encoded in $\nu = f(x)$

Spatial encoding gradient pulses part of pulse sequence

Example:

phase encode y,
frequency encode x



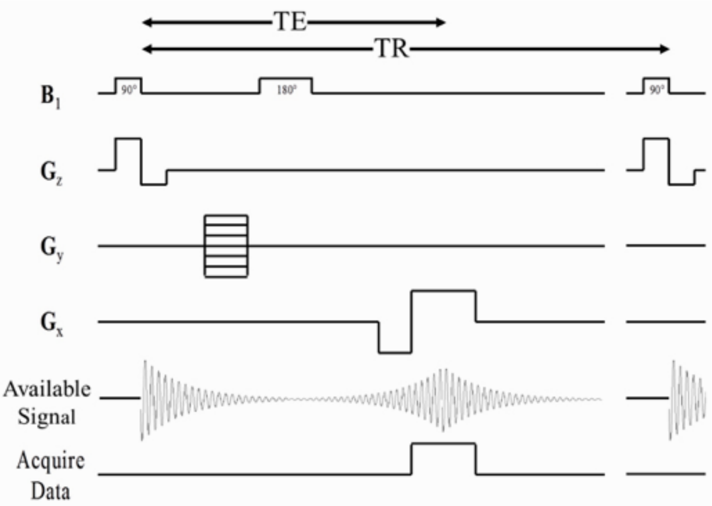
Phase- and frequency-encoding pulses part of a longer pulse sequence that repeats with period TR

At each repeat the amplitude of G_y, the phase-encoding pulse, has a different amplitude (as indicated on the figure)

For example:

- 1^{st} iteration of sequence: $G_y = 0$;
- 2^{nd} iteration of sequence: $G_y = +\eta$;
- 3^{rd} iteration of sequence: $G_y = -\eta$;
- ...

Example pulse sequence



Example of spin-echo pulse sequence

Data is acquired at spin-echo time as shown

Combination of phase and frequency encoding pulses and repetition to obtain N_y data points completes ones transverse slice

Summary of section 1

Field gradient makes Larmor *frequency* a function of position;

Phase difference as a function of position develops during application of gradient pulse

Exploit the position dependence of frequency and phase to encode image in k-space

Pulse sequences designed to optimise contrast within slice for various tissues and types of investigation

Section 2

Image reconstruction in MRI

Reconstruction of the MR image

Gradient pulses G_i are used to allow slice-selective excitation and to allow spatial information to be encoded into the net magnetisation:

$$B_z(x, y, z, t) = B_0 + xG_x(t) + yG_y(t) + zG_z(t)$$

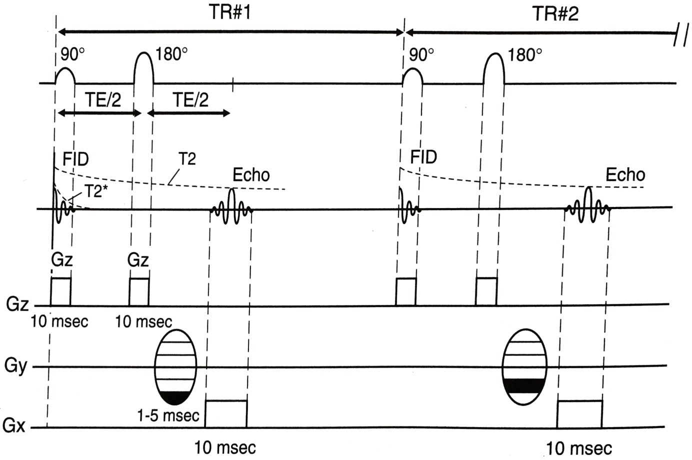
Spatial information is encoded into net magnetisation in k-space, often:

- Frequency encoding is used to encode features in the x direction
- Phase encoding is used to encode features in the y direction

2D Fourier transform used to transform image in k space to image in coordinate space

Pulse sequence is repeated to collect data for all $N_{k_x} \times N_{k_y}$ pixels of image

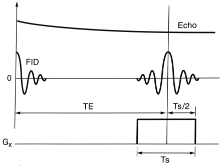
Spin-echo sequence



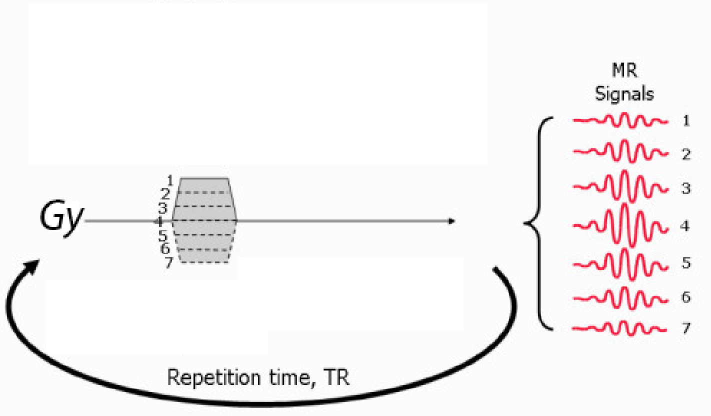
Readout occurs when frequency-encoding pulse (G_x) is on (T_S, the sampling time)

Each repetition corresponds to a new G_y, i.e. a new encoding of phase

Take N_{k_y} repetitions to fill N_{k_y} rows in the image



Phase encoding (1 of 3)

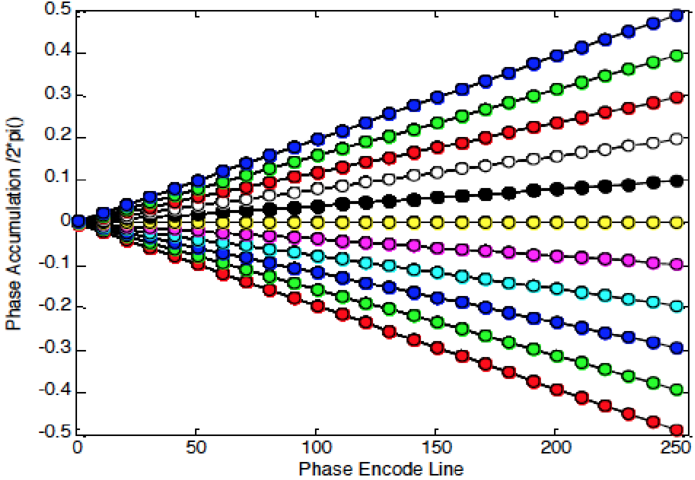


Central line in k-space will contain phase-encoding step with weakest gradient and strongest signal

Periphery of k-space sill contain phase-encoding steps with the strongest gradients and weakest signal

Each slice “has its own k-space because excitation is tuned to G_z

Phase encoding (2 of 3)



Phase wrt $y = 0$ accumulates with time, t, while phase-encoding gradient, G_y, is on:

$$\Phi(G_y, y, t) = (\gamma G_y)\, yt \qquad (1)$$

The slope of the line in the figure is determined by y

I.e. the rate of change of phase (frequency) is given by:

$$\frac{\Delta\Phi}{\Delta t} = (\gamma G_y)\, y$$

2D Fourier transform revisited

If the phase-encoding pulse is of length τ_{pe}, then the change of phase of the spins relative to $y = 0$ at the end of the pulse will be given by:

$$\frac{\Delta\Phi}{\Delta y} = (\gamma G_y \tau_{pe})$$

Lets take the start of the frequency-encoding pulse, G_x, to be at $t = 0$, then, the phase advance of ^{1}H nuclei at x after time t will be:

$$\phi(G_x, x, t) = (\gamma G_x)xt \qquad \text{i.e.} \qquad \frac{\Delta\phi}{\Delta x} = (\gamma G_x)t$$

As in lecture 10, let $\rho(x, y)$ be the intensity pixel-by-pixel in coordinate space, then the signal S will be given by:

$$S\left(G_y, \tau_{pe}, G_x, t\right) = \int_{y_{\min}}^{y_{\max}} \int_{x_{\min}}^{x_{\max}} \rho\left(x, y\right) \exp\left[-i\left(\gamma G_x t\right) x\right] \exp\left[-i\left(\gamma G_y \tau_{pe}\right) y\right] dxdy$$

2D Fourier transform revisited

In lecture 10, 2D Fourier transform from coordinate to k space was given as:

$$S\left(k_x, k_y\right) = \int_{y_{\mathrm{min}}}^{y_{\mathrm{max}}} \int_{x_{\mathrm{min}}}^{x_{\mathrm{max}}} \rho\left(x, y\right) \exp\left(-i2\pi k_x x\right) \exp\left(-i2\pi k_y y\right) dxdy$$

where $S(k_x, k_y)$ is the intensity pixel-by-pixel in k space

If we identify:

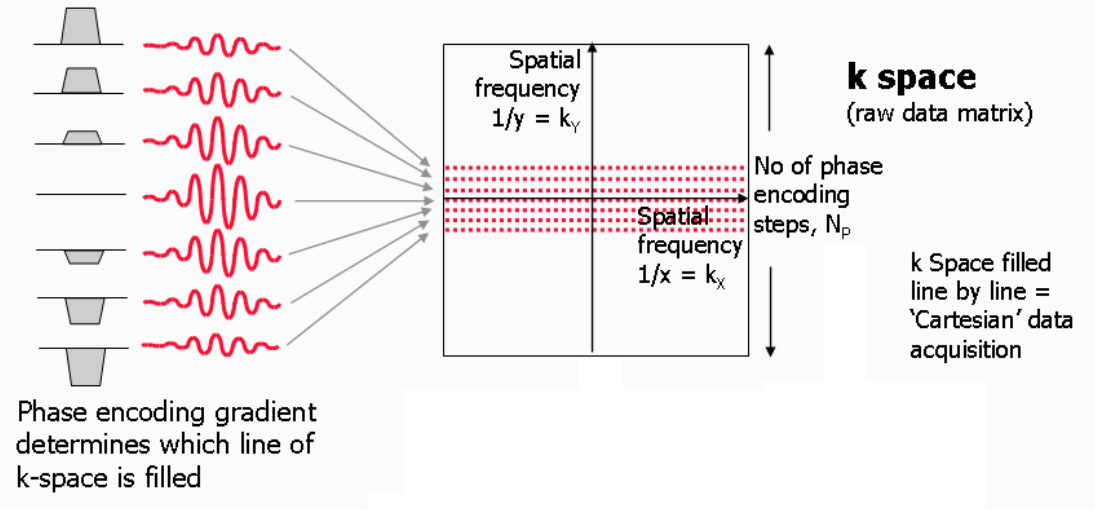
$$\begin{aligned} k_x &= \frac{\gamma}{2\pi} G_x t \\ k_y &= \frac{\gamma}{2\pi} G_y \tau_{pe} \end{aligned}$$

then:

$$S\left(k_x, k_y\right) = S\left(G_y, \tau_{pe}, G_x, t\right)$$

And the measured signal, S, is the k-space representation of the coordinate-space intensity ρ

Phase encoding (3 of 3)



Summary of section 2

The slice is selected by tuning the RF frequency and G_z

The k_x coordinate is obtained from frequency encoding **at readout**

The k_y coordinate is obtained from phase encoding **“passively” by manipulating phase during free induction decay (FID)**

Section 3

Images and artefacts

Artefacts in MRI

Ghosting due to total internal reflection of bright sources in optical photography

Just as in optical photography, artefacts are unwanted image features

Artefacts arise from many causes:

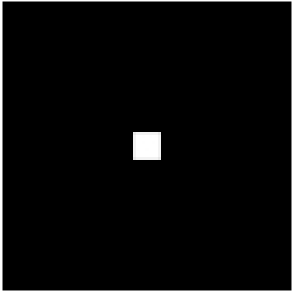
- Field imperfections (not addressed below)
- Movement of patient or organ
- Magnetic material (e.g. from bone repairs)
- Chemical composition uncertainties

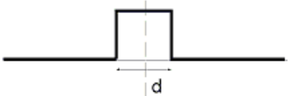
My objective is to give examples, there is an extensive literature on the subject

Trial: square centred at $(x, y) = (0, 0)$

Consider spatial encoding, x, direction

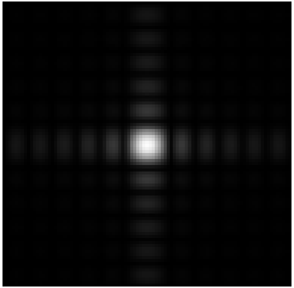
Coordinate space

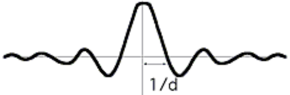




$$\rho(x) = \begin{cases} \rho_0 & \text{for } -\frac{d}{2} < x < \frac{d}{2} \\ 0 & \text{otherwise} \end{cases}$$

k space





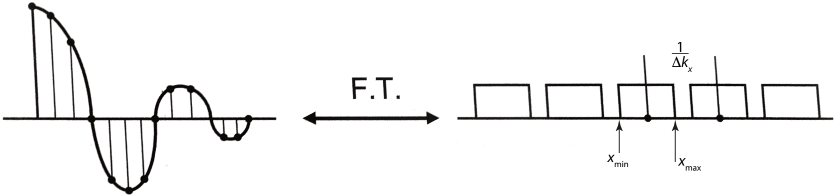
$$S(\Delta k_x) = S'_0 \mathrm{sinc}(\Delta k_x)$$

Sampling of the signal recorded along k_x

The Fourier transform of "sinc" function will give "box" function if all Δk_x are sampled

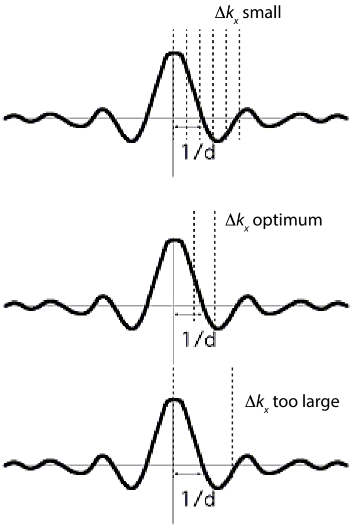
But, sinc function is only sampled at intervals of Δk_x

This means that Fourier transform of the sampled sinc function generates a series of (distorted) images of the box:



Sequence truncated at field of view

Sampling of the signal recorded along k_x



Nyquist theorem:
To reconstruct a bandwidth limited signal, require to sample the highest frequency that the signal contains at least twice

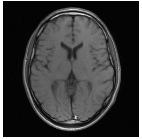
In our case, the bandwidth is limited by the truncation of the sinc function by the field of view in k space

At limit of resolution, box functions are "just separated"

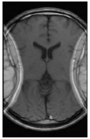
If sampling rate is too low, $\Delta k_x > \frac{2}{2\pi(x_{\max} - x_{min})}$), the boxes overlap and aliasing occurs

Aliasing (wraparound)

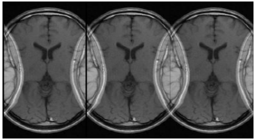
Image of head with appropriate sampling rate (field of view)



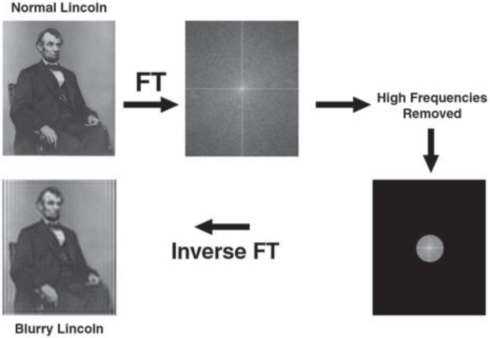
Truncated field of view (in *k*-space) yields sampling rate that is too low . . .

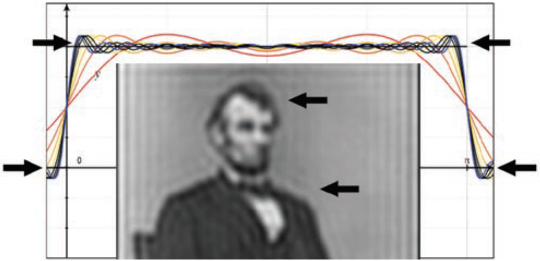


. . . and leads to aliasing



Effect of truncation in k space

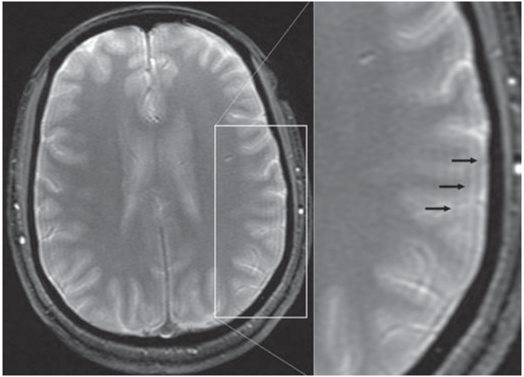




The Gibbs phenomenon

Artefact occurs at interfaces between tissues which have a rapid change in signal . . . "high contrast interfaces"

E.g. skull to brain



Summary of section 3

Nyquist theorem:

For accurate reconstruction of features require to sample the highest frequency at least twice

If the sampling rate is too low, aliasing (or wraparound) artefacts occur

Truncation of sinc function by field of view in k-space can lead to image blur and the "Gibbs phenomenon" at places where there is a rapid change in image brightness

Section 4

Random motion artefacts

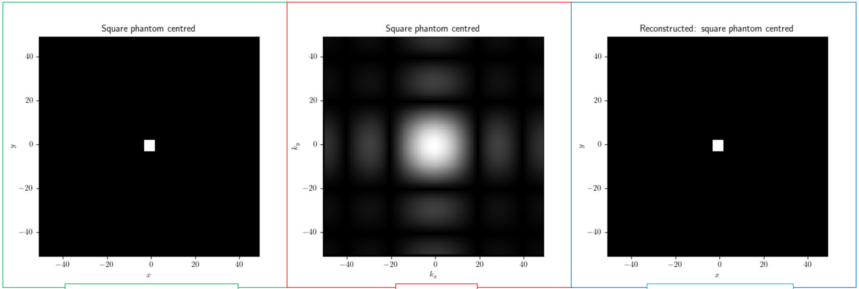
Motion artefacts; general comments

Motion artefacts most commonly observed in the phase-encoding direction

This is because:

- Motion along any field gradient results in the abnormal integration of phase, which is then incorrectly mapped onto the phase-encoding direction
- Frequency encoding is performed while G_x gradient pulse is on; typically for around 10 ms. Only very modest displacements can occur in such a short time. The result is that random displacements in the frequency-encoding direction lead to blur
- By contrast, displacements due to motion can build up between phase-encoding pulses, G_y, as these occur at intervals of TR . . . and TR can range from, e.g. 500 ms to a second or so

Square phantom at the centre of the field in coordinate space

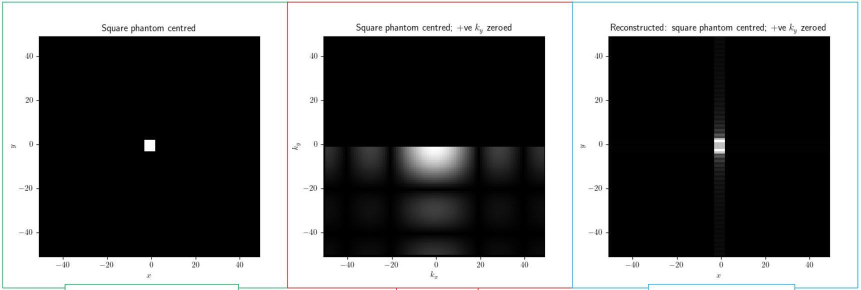


Coordinate space | *k* space | Reconstructed

Fourier transform of a 2D real function yields is Hermitian, so, component at k_i is the complex conjugate of the component at $-k_i$.

This means there is no information in quadrant $(-k_x, -k_y)$ that is not in quadrant (k_x, k_y) and the information content of quadrants $(-k_x, k_y)$ and $(k_x, -k_y)$ is the also the same

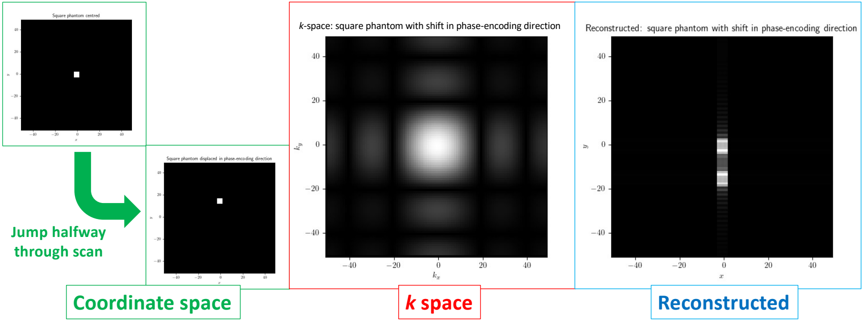
Loss of half of phase-encoding in k space



Coordinate space **k space** **Reconstructed**

Loss of information in "second half" of phase-encoding cycle leads to loss of definition/banding in the phase-encoding direction

Effect of sudden displacement in phase-encoding direction

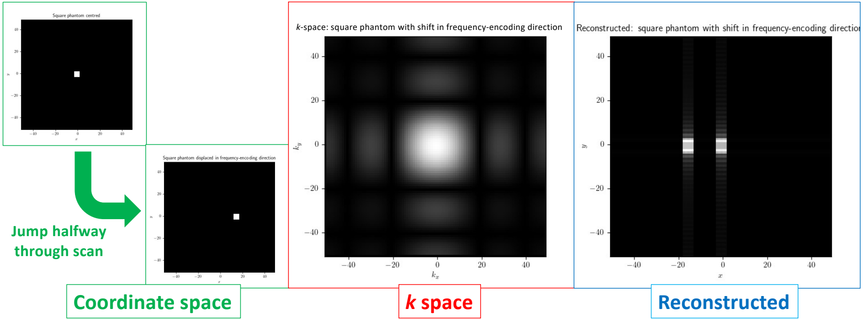


Consider that square phantom jumped as shown halfway through the cycle

The "bottom half" of k-space arises from the original position, the "top half" from the new position

The result multiple images and ghosting in the phase-encoding direction

Effect of sudden displacement in frequency-encoding direction

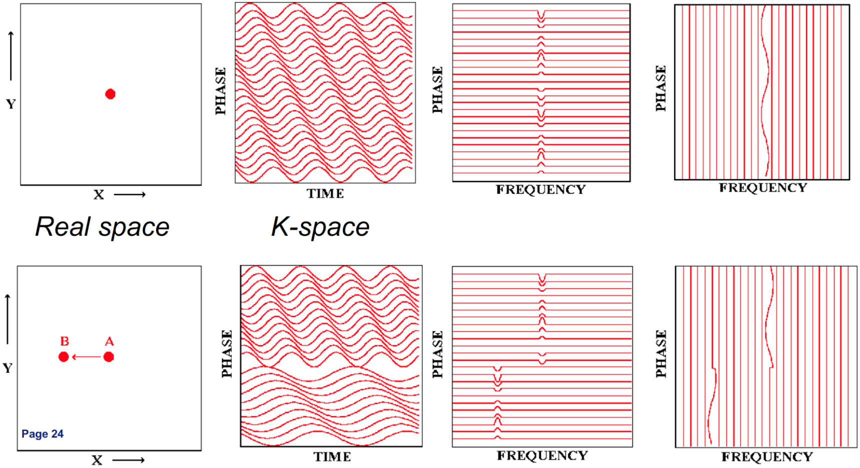


Consider that square phantom jumped as shown halfway through the cycle

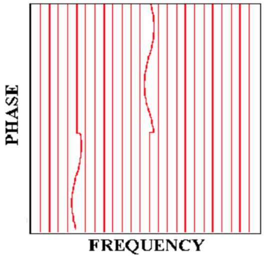
Again, the "bottom half" of k-space arises from the original position, the "top half" from the new position

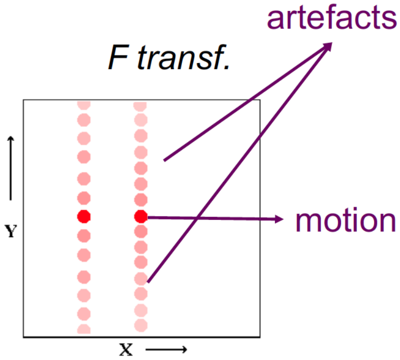
The result multiple images and ghosting in the frequency-encoding direction

Another way of looking at displacement along the frequency-encoding direction



Another way of looking at displacement along the frequency-encoding direction

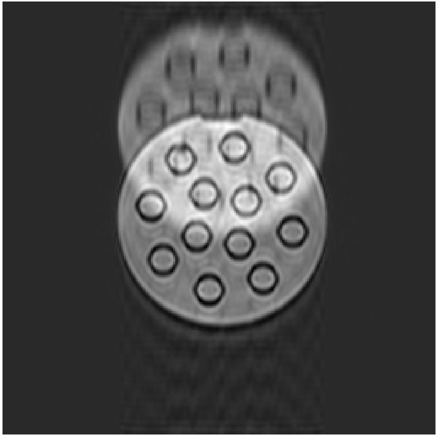




Random motion artefacts

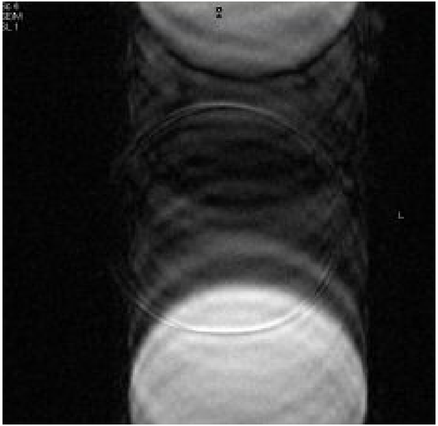
Displacement artefacts: questions 1

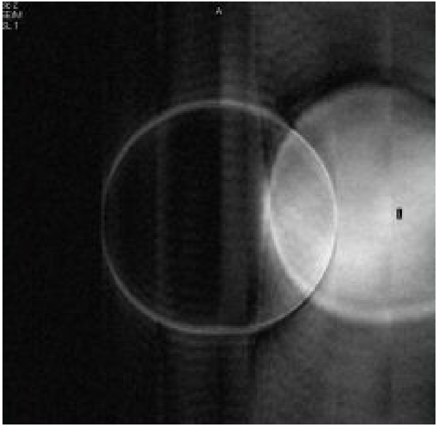
What causes the artefacts seen in the following images?



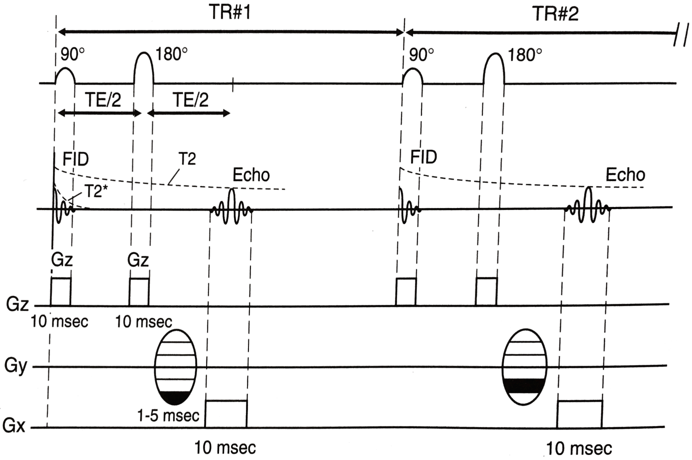
Displacement artefacts: questions 2

What causes the artefacts seen in the following images?





Spatial encoding, reprise



Signal, S: $S = S(G_x, t, G_y, \tau_{pe})$

Frequency encoding in x direction:

$$\phi(G_x, x, t) = (\gamma G_x x)\, t$$

Phase encoding in y direction:

$$\Phi(G_y, y, \tau_{pe}) = (\gamma G_y y)\, \tau_{pe}$$

Transformation to k space:

$$k_x = \frac{\gamma}{2\pi} G_x t$$
$$k_y = \frac{\gamma}{2\pi} G_y \tau_{pe}$$

$$S\left(G_y, \tau_{pe}, G_x, t\right) = S(k_x, k_y) = \int_{y_{\min}}^{y_{\max}} \int_{x_{\min}}^{x_{\max}} \rho\left(x, y\right) \exp\left[-i\left(\gamma G_x t\right) x\right] \exp\left[-i\left(\gamma G_y \tau_{pe}\right) y\right] dxdy$$

Periodic motion; overview

Organs that undergo periodic motion include the heart, aorta, ...

Frequency encoding takes place over a period of $\sim 10\,\text{ms}$ when the G_x pulse is on. This corresponds to a frequency of $100\,\text{Hz}$; i.e. 100 cycles *per second*. Such rapid oscillations are not present in the body. Oscillations at the frequency of the heart beat, for example, lead to only small excursions while G_x is on and so lead to minor loss of detail in the image

The process of phase encoding requires multiple (N_y) repetitions to complete. While the G_y pulse itself is short, it is repeated at time intervals equal to TR

The time period relevant for phase encoding, therefore, is TR. A typical value for TR is $500\,\text{ms}$, corresponding to a frequency of $2\,\text{Hz}$. Many structures in the body, for example the heart, execute periodic motion with period comparable to TR

Periodic-motion artefacts, therefore, occur in the phase-encoding direction

Periodic motion artefact

The phase, Φ, used for spacial encoding in the phase-encoding direction is given by:

$$\Phi(G_y, y, \tau_{pe}) = (\gamma G_y y)\, \tau_{pe}$$

If the position, y of a feature undergoes periodic motion, then:

$$y \to y' = y + d_0 \sin \omega_{pma} t$$

And so the phase that enters the phase-encoding equation becomes a function of the "periodic motion artefact" frequency ω_{pma}:

$$\Phi(G_y, y, \tau_{pe}) \to \Phi'(G_y, y, \tau_{pe}, \omega_{pma}) = (\gamma G_y y')\, \tau_{pe} = 2\pi k_y y' = 2\pi k_y\, (y + d_0 \sin \omega_{pma} t)$$

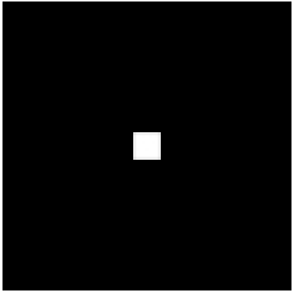
Addition of phase, leads to displacement in k space

Periodic motion artefact

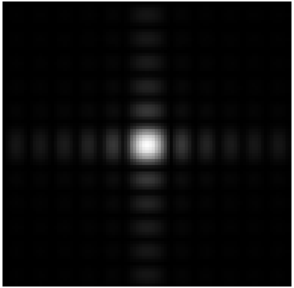
In considering the impact of the additional phase added by periodic motion, we must remember that coordinate space is represented across the k space

Consider again the square at the centre of coordinate space

Coordinate space



k space

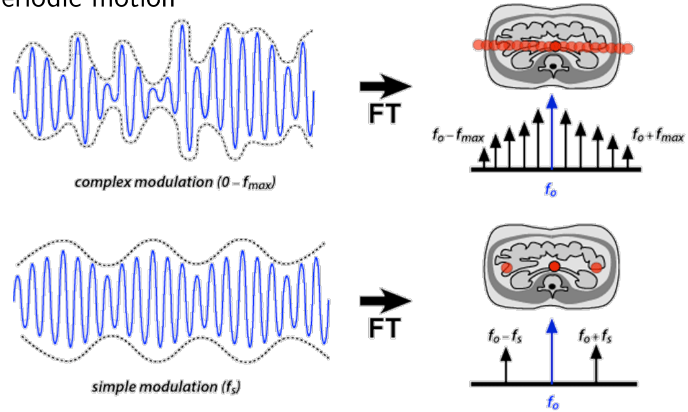


The result of the additional phase is to shift the whole pattern in k space

Periodic motion artefact

For a complete treatment, we need to look at the impact of Φ' on the encoding equation ...

Instead, lets consider the modulation of the phase-encoding pattern that results from the periodic motion



The amplitude of the periodic shift in the y direction generated by the periodic motion is given by:

$$\delta y = \frac{\mathrm{TR}}{\tau_{pma}} [y_{\max} - y_{\min}]$$

Periodic motion: breathing and heart beat

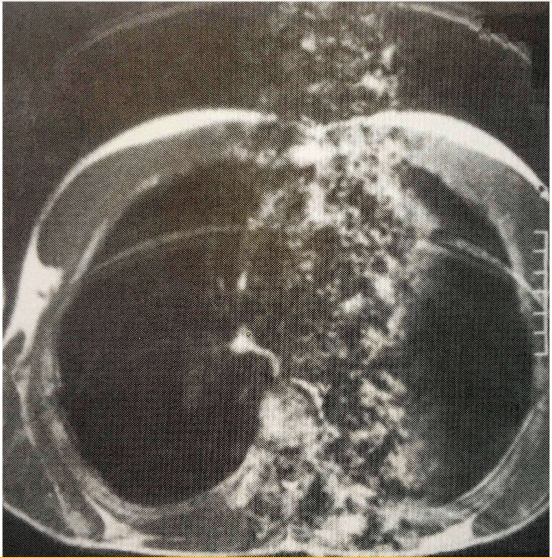


Image of chest showing ghosting arising from breathing and heart beat

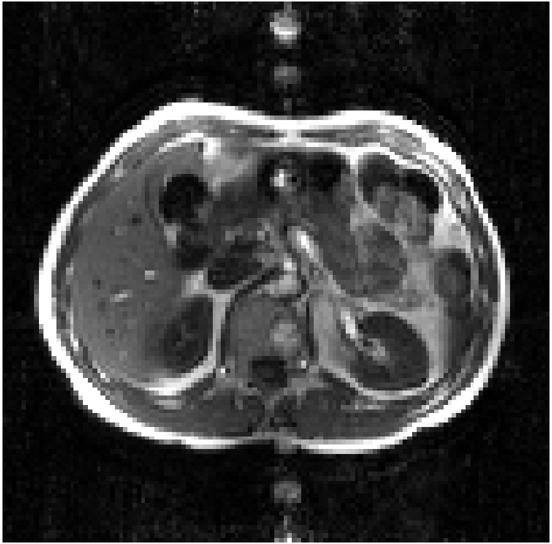
Respiratory motion causes a number of distinct images of the chest wall

Cardiac motion, more complex and multi-facetted, results in the column of overlaping images to the right of centre

In general, the more rapid the motion, the more widely spread will be the ghosts:

$$\delta y \propto \frac{1}{\tau_{pma}}$$

Periodic motion: problem



The periodic motion artefact in the image facing is caused by the periodic motion of a small region of the scan plane The imaging

parameters that were used were TE$=40\,\text{ms}$ and TR$=100\,\text{ms}$

1. Estimate the period of the movement from the separation of the ghosts (assume that the field of view is $40\,\text{cm}$).
2. What structure in the body might give rise to this repeating feature?
3. Identify the position of the primary source of the artefact in the image.

Answer will be given in the answers to the second problem sheet.

Summary of section 4

Random motion causes:

- Since frequency-encoding happens rapidly, most often motion in frequency-encoding causes blur
- Gross sudden movements in frequency-encoding direction can lead to multiple images and ghosting
- Phase encoding builds up over many cycles, so, motion in phase-encoding direction often leads to multiple images and ghosting

Periodic motion artefacts occur in the phase-encoding direction

- Complex or rapid motion leads to complex modulation of the phase-encoding pattern; simple or slow motion leads to simple modulation of the phase-encoding pattern
- Periodic motion artefacts can manifest as ghost images displaced in the phase-encoding direction (e.g. breathing) or complex patterns (e.g. heart motion)