Physics of Medical Imaging and Radiotherapy
Magnetic Resonance Imaging

Lecture 1: Introduction to MRI and quantum mechanical foundations

1 Introduction

Magnetic resonance imaging (MRI) exploits the magnetic moment of nuclei within soft tissues to generate an image. In contrast to methodologies that exploit ionising radiation, MRI does not directly induce DNA damage or produce ions that may indirectly cause damage.

Nuclei with non-zero spin have a magnetic moment. A strong, uniform magnetic field can be used to align the spins. In the ground state, the spin, and therefore the nuclear magnetic moment is aligned parallel to the magnetic field. The uniform magnetic field provides an axis along which the spin vector can be quantised. As a result, the spin vector may exist in one of a limited number of states, each state placing the nuclear magnetic moment in a particular orientation. Electromagnetic radiation may be used to excite transitions between the various spin states. And, the electromagnetic radiation emitted when a nucleus raised to an excited state relaxes to the ground state may be detected.

The most abundant nucleus in tissue is hydrogen. The half-integer spin of a proton makes it an ideal vehicle for the generation of a signal from which an image can be generated. The uniform magnetic field in an MRI scanner is usually generated using a superconducting solenoid that produces a magnetic field in the range $\sim 0.5\,\mathrm{T}$ to $\sim 3\,\mathrm{T}$. The energy splitting between a proton with spin aligned parallel to the magnetic field and one with spin anti-parallel corresponds to electromagnetic radiation in the radio-frequency range $\sim 10\,\mathrm{MHz}$ to $\sim 100\,\mathrm{MHz}$. The genius of the MRI technique is the manipulation of magnetic fields to adjust the response of tissue to the exciting radio waves and the subsequent detection of the signals generated as the nuclei relax to their ground states.

My aim is that, by the end of the MRI section of the course, you will have an understanding of the principles exploited to generate an image in an MRI scanner.

2 Coordinate system

The conventional definition of the coordinate system in an MRI scanner, together with the naming convention for the three orthogonal planes, is shown in figure 1. The coordinate system is such that its z axis lies along the direction defined by $\mathbf{B}_0$ and is horizontal. Since the principal uniform magnetic field is created using a solenoid, the z axis is also the axis of the principal solenoid. The y axis is taken to point vertically upwards and the x axis is defined to complete a right-handed coordinate system.

3 Quantum mechanical foundations

The magnetic moment, $\boldsymbol{\mu}$, of a proton with spin vector $\mathbf{s}$ is given by:

$$\boldsymbol{\mu} = \gamma \mathbf{s} \, ;$$

where γ is the gyromagnetic ratio. If the proton is immersed in a magnetic field $\mathbf{B}_0$, then the Hamiltonian describing the proton is perturbed by a contribution $\delta\mathcal{U}$ given by:

$$\delta\mathcal{U} = -\mathbf{B}_0 \cdot \boldsymbol{\mu} \, .$$

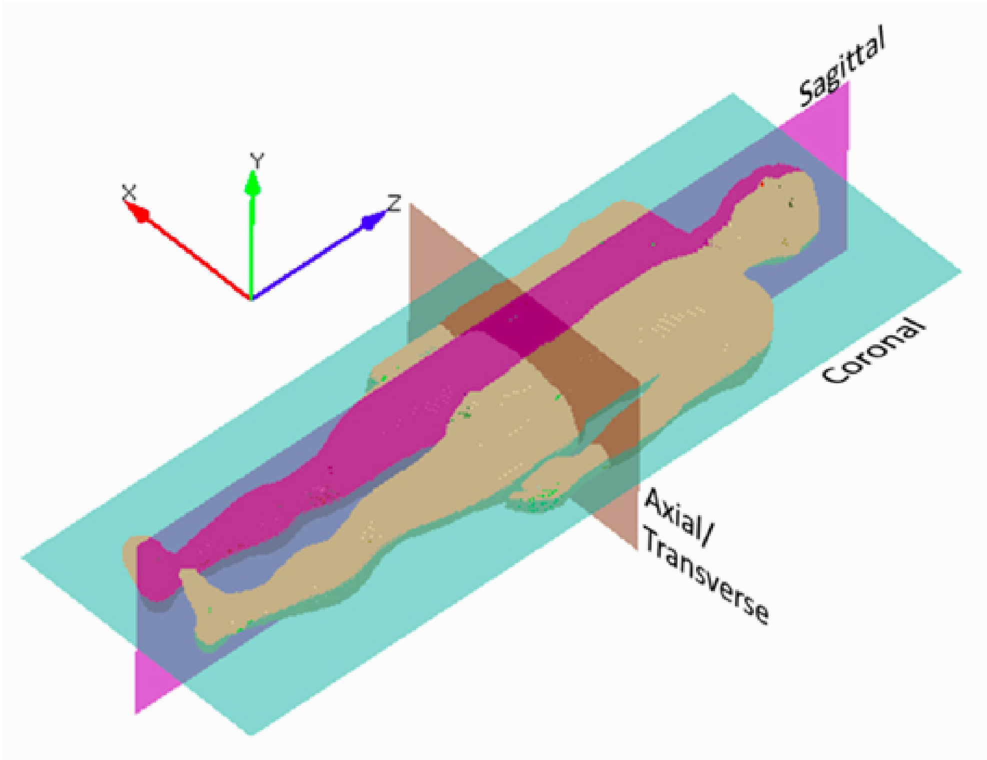

Figure 1: Conventional terminology and orientation of planes in and MRI system. The axes of the conventional, right handed Cartesian coordinate system are also shown.

The result of the perturbation is to split the proton energy level such that, if the unperturbed energy of the proton was E, then, the perturbed energy E' is given by:

$$E' = E \pm E_{m_s}\,;$$

where:

$$E_{m_s} = -m_s \gamma \hbar B_0\,;$$

and m_s is the quantum number associated with the component of the proton spin parallel to $\mathbf{B}_0$, $\hbar$ is Planck's constant divided by 2π, and B_0 is the magnitude of $\mathbf{B}_0$. Since the proton has half-integer spin, $m_s = \pm\frac{1}{2}$, so that, ΔE, the splitting between the two energy levels, is given by:

$$\Delta E = \gamma \hbar B_0\,.$$

The angular frequency of the photon emitted or absorbed in transitions between the two energy states, ω_0, is given by:

$$\Delta E = \hbar \omega_0\,;$$

and therefore:

$$\omega_0 = \gamma B_0\,.$$

The linear dependence of ω_0 on B_0 is referred to as the Larmor equation and ω_0 is the Larmor frequency.

4 Bulk magnetisation

In the absence of a magnetic field, the populations of hydrogen nuclei in the $m_s = +\frac{1}{2}$ and $m_s = -\frac{1}{2}$ states is the same. In the presence of the magnetic field, the $m_s = +\frac{1}{2}$ state is populated at the expense of the $m_s = -\frac{1}{2}$ state.

The ratio of the number of nuclei in the lower energy $m_s = -\frac{1}{2}$ state, N_-, to the number in the $m_s = +\frac{1}{2}$ state, N_+, is given by:

$$\frac{N_+}{N_-} = \exp\left(-\frac{\Delta E}{k_B T}\right)$$

where k_B is Boltzmann's constant, and T is the temperature. For the human body, $k_B T \approx 25.7\,\mathrm{meV}$, so, for the magnetic fields relevant to an MRI scanner:

$$\Delta E << k_B T\,.$$

Therefore, the exponential may be expanded to yield:

$$N_+ - N_- \approx N_- \frac{\Delta E}{k_B T} \approx N_S \frac{\Delta E}{2 k_B T}$$

where $N_S = N_+ + N_- \approx 2N_- \approx 2N_+$ is the number of "available spins", i.e. the number of hydrogen nuclei. For the human body, the ratio of the difference $N_+ - N_-$ to N_s for tissue exposed to a field $B_0 = 1.5\,\mathrm{T}$ is approximately given by:

$$\begin{aligned}\frac{N_+ - N_-}{N_S} &\approx \frac{42.58 \times 10^6 \times 6.6 \times 10^{-34} \times 1.5}{2 \times 1.38 \times 10^{-23} \times 300} \\ &\approx 5 \times 10^{-6}\,.\end{aligned}$$

This means that only 5 in every 10^6 protons contributes to the magnetisation. Fortunately, Avogadro's number is very large, and 1 g of water contains approximately 10^{22} hydrogen nuclei. As a result, the net bulk magnetisation produced by the external magnetic field applied to tissue is measurable.

5 Classical derivation of the Larmor equation

The bulk magnetisation $\mathbf{M}$ is the result of summing the contributions of the magnetic moments of each hydrogen nucleus. The magnetic moment of each hydrogen nucleus makes a small angle to the quantisation axis. As a result, the nuclear magnetisations precess at the Larmor frequency around the applied magnetic field direction. The net magnetisation is the sum of the contributions of all the nuclear magnetic moments. The components of the nuclear magnetic moments in the plane perpendicular to $\mathbf{B}_0$ are orientated in random directions in the (x, y) plane and therefore sum to zero. In contrast, the components parallel to $\mathbf{B}_0$ sum to produce the net magnetisation $\mathbf{M}$. In the equilibrium ground state, $\mathbf{M}$ is parallel to $\mathbf{B}_0$.

If the net magnetisation vector is rotated away from the $\mathbf{B}_0$ direction, the Lorentz force then produces a torque on the magnetisation vector given by $\mathbf{M} \times \mathbf{B}_0$ which causes $\mathbf{M}$ to precess around $\mathbf{B}_0$. The rate of precession is given by:

$$\frac{d\mathbf{M}}{dt} = \gamma\left(\mathbf{M} \times \mathbf{B}_0\right)\,.$$

This equation is referred to as the "Bloch equation". If $\mathbf{M}$ makes an angle θ w.r.t. $\mathbf{B}_0$, then:

$$\mathbf{M} \times \mathbf{B}_0 = (M B_0 \sin\theta)\,\hat{\boldsymbol{\omega}}_\mathbf{0}\,.$$

So:

$$\frac{d\mathbf{M}}{dt} = (\gamma M B_0 \sin\theta)\,\hat{\boldsymbol{\omega}}_\mathbf{0} = (M \omega_0 \sin\theta)\,\hat{\boldsymbol{\omega}}_\mathbf{0}\,;$$

which gives the Larmor equation:

$$\omega_0 = \gamma B_0\,.$$

The Larmor frequency, ω_0, can now be interpreted as the rate at which the bulk magnetisation precesses around the external magnetic field vector. It is no accident that the Larmor equation derived from the consideration of the splitting of the quantum-mechanical energy levels is the same as that derived from the, classical, consideration of the precision of the bulk magnetisation. The bulk magnetisation is produced as a coherent effect of many hydrogen nuclei. The magnetic fields of each of these nuclei is exposed to a torque and therefore precesses at a rate determined by the Larmor equation.

6 Rotation of the bulk magnetisation

Consider a magnetic field $\mathbf{B}_1$ applied in the field perpendicular to $\mathbf{B}_0$ (i.e. $\mathbf{B}_1$ is such that $\mathbf{B}_1 \cdot \mathbf{B}_0 = 0$). The effect is to produce another torque on $\mathbf{M}$ that will cause it to undergo a second precession around the axis defined by $\mathbf{B}_1$.

The field $\mathbf{B}_1$ is produced in an MRI scanner using coils which create a magnetic field oscillating at the Larmor frequency, ω_0; ω_0 is in the radio frequency range. $\mathbf{B}_1$, then, lies in the (x, y) plane and may be created with components along the x and y directions given by:

$$B_{1_x} = B_1 \cos(\omega_0 t + \alpha) \text{ and}$$
$$B_{1_y} = B_1 \sin(\omega_0 t + \beta)\,;$$

where α and β are phase parameters and t is time. Changing basis, the oscillating field can be written in terms of two counter-rotating circularly polarised fields; one field, $\mathbf{B}_{1_{ac}}$, rotating anticlockwise about the z axis with a phase ϕ_{ac} and one, $\mathbf{B}_{1_c}$, rotating clockwise about the z axis with a phase ϕ_c. The magnitudes of the circularly polarised field are equal:

$$B_{1_{ac}} = B_{1_c} = \frac{B_1}{2}\,;$$

and the phases evolve with time according to:

$$\phi_{ac} = \omega_0 t + \alpha'\,; \text{and}$$
$$\phi_c = -\omega_0 t + \beta'\,;$$

where α' and β' are constant phase parameters.

The magnetic field vector of one of the two circularly polarised fields will be rotating in the same direction as the bulk magnetisation. Lets call the field that is rotating in the same direction $\mathbf{B}_{\text{lock}}$. If the angular frequency of the rotating fields is the same as the Larmor frequency then $\mathbf{B}_{\text{lock}}$ will appear stationary from the point of view of the precessing bulk magnetisation. The second of the two counter rotating fields will appear to be rotating with an angular velocity of twice the Larmor frequency. Its effect on the net magnetisation will rapidly average to zero. The net result, then, is that the magnetisation vector will begin to precess around $\mathbf{B}_{\text{lock}}$. The effect of the RF $\mathbf{B}_1$ pulse is shown schematically in figure 2. In the laboratory frame of reference, the net magnetisation is initially along the z axis. During the RF $\mathbf{B}_1$ pulse the magnetisation vector is rotated away from the z axis and "spirals down" towards the (x, y) plane. In a frame rotating at an angular velocity ω_0 in the same direction as the precession (the rotating frame of reference), $\mathbf{B}_{\text{lock}} = \mathbf{B}^+$ appears stationary and the net magnetisation appears simply to rotate away from the z axis.

Since the magnitude of $\mathbf{B}_1$ is small compared to the principal, uniform field, $\mathbf{B}_0$, the rate of precession around $\mathbf{B}_{\text{lock}}$ will be much smaller the rate of precession around $\mathbf{B}_0$. By controlling the length of time for which the field $\mathbf{B}_1$ is applied it is possible to control the magnitude of the angle by which the magnetisation vector is rotated away from the z axis.

The full calculation of the rate of precession of the net magnetisation induced by the RF B_1 pulse requires the integration of the motion induced by the effective time-varying magnetic field generated by the combination of $\mathbf{B}_0$ and $\mathbf{B}_1$. Before the RF B_1 pulse is applied, the magnetisation is stationary in the frame rotating at ω_0 in the same direction as the Larmor precession (the co-rotating frame). Since the magnetisation is stationary it is not

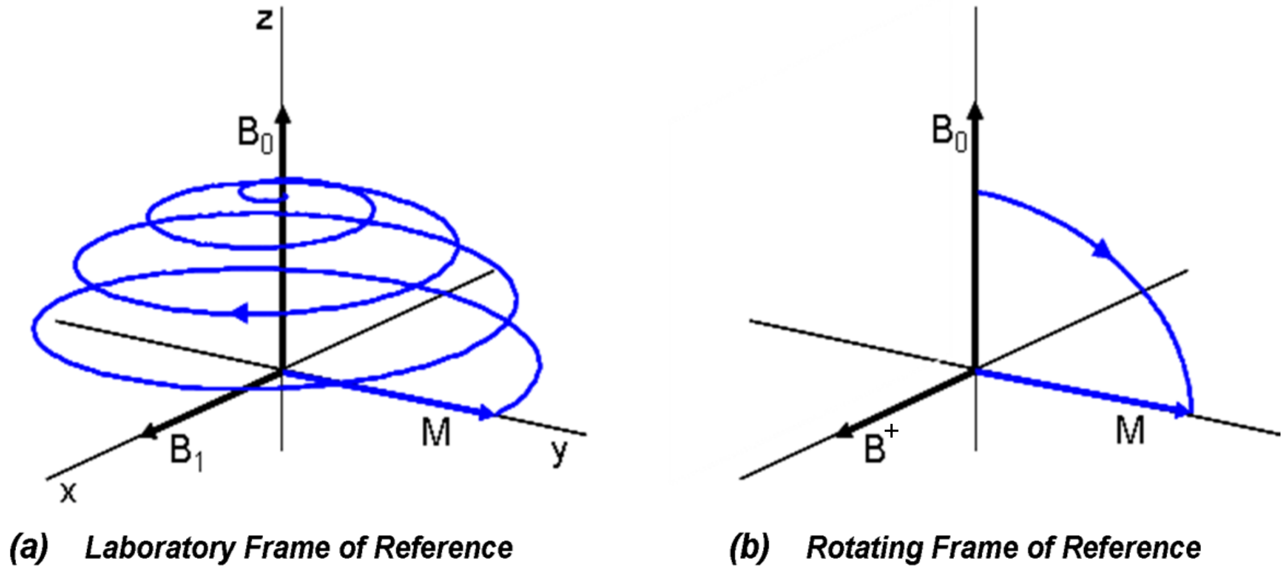

Figure 2: Schematic representation of the effect of the RF $\mathbf{B}_1$ pulse: (a) in the "Laboratory Frame", i.e. in the frame in which the scanner is at rest; and (b) in the "Rotating Frame", which is rotating at an angular velocity ω_0 in the same direction as the precession.

experiencing a torque and therefore the effective magnetic field may be taken to be zero. When the RF B_1 pulse is turned on, the magnetisation will experience a torque generated by the field B_1, in the (x, y) plane, that appears stationary in the co-rotating frame. As a result, the rate of precession about $\mathbf{B}_{\text{lock}}$, ω_1, in the rotating frame is given by:

$$\omega_1 = \gamma B^+ = \gamma B_1 \, .$$

The RF $\mathbf{B}_1$ field is applied in a pulse of duration t_P. The flip angle, α, the angle through which the magnetisation is rotated, is proportional to the magnitude and duration of the RF $\mathbf{B}_1$ pulse:

$$\alpha = \omega_1 t_P = \gamma B_1 t_P \, .$$