Physics of Medical Imaging and Radiotherapy
Magnetic Resonance Imaging

Lecture 2: Generation of contrast

1 Free induction decay

1.1 Generation of signal: precession of magnetisation vector

The "headlines" of the MRI process introduced so far are:

1. Apply the principal uniform, static, magnetic field, $\mathbf{B}_0$ with magnitude M_0, to create the net magnetisation, $\mathbf{M}$, aligned with the principal magnetic field. The z axis of a right-handed coordinate system is defined to lie along the direction defined by $\mathbf{B}_0$.
2. Apply a pulse of a magnetic field, $\mathbf{B}_1$, directed in the (x, y) plane. The RF $\mathbf{B}_1$ pulse oscillates at a frequency, ω_0, chosen to match the Larmor frequency at which the magnetisation is precessing around the z axis. The length of time for which the RF $\mathbf{B}_1$ pulse is applied, $\tau_{\rm RF}$, is chosen such that at the end of the pulse the magnetisation has been rotated through an angle α. The angle α is referred to as the "flip" angle.

If the flip angle is not a multiple of $180°$, then, the result of the B_1 pulse to produce a component of magnetisation in the (x, y) plane that processes at the Larmor frequency, ω_0. Since a changing magnetic field is a source of electromagnetic radiation, the rotating magnetisation radiates an RF wave at the Larmor frequency.

1.1.1 Free induction decay

Consider the case when the application of a 90° pulse has rotated the net magnetisation vector into the (x, y) plane so that the z component of $\mathbf{M}$ is zero. The potential energy of the system, $-\mathbf{M} \cdot \mathbf{B}_0$, is larger than it is in the equilibrium configuration, and therefore the magnetisation vector relaxes back towards the equilibrium state in which it is aligned with the z axis. The rate at which M_z recovers is determined by the difference between the value at equilibrium, M_0, and the instantaneous value M_z; $M_z \leq M_0$. Observations of the rate of decay of the amplitude of the RF signal indicate that the recovery of the equilibrium magnetisation is characterised by a time constant, T_1, and that the recovery of the longitudinal magnetisation is given by:

$$\frac{dM_z}{dt} = \frac{M_0 - M_z}{T_1} \, .$$

Therefore:

$$M_z(t) = M_0 \left[1 - \exp\left(-\frac{t}{T_1} \right) \right] \, .$$

The recovery of the equilibrium magnetisation is referred to as "free induction decay".

1.1.2 Spin-lattice relaxation

At the microscopic, i.e. the nuclear, level, the relaxation of the net magnetisation is caused by transitions to the lower-energy parallel state by hydrogen nuclei that had been excited to the anti-parallel spin state. These spin transitions release energy into the material, i.e. into the "lattice", so the relaxation process is referred to as "spin-lattice relaxation". The coupling of the motion of the hydrogen nuclei to the lattice is relatively ineffective. As a result, the time constant, T_1, is large. Typically, $T_1 > 200\,$ms.

1.1.3 Spin-spin relaxation

While the spin-lattice coupling is not very strong, the coupling of one nuclear magnetic moment to the fields generated by adjacent nuclear spins and the local magnetic environment is strong. This strong "spin-spin" coupling gives rise to a mechanism that causes the long-range coherence of the nuclear magnetic moments to decay. Since the spin-spin interaction is strong, the rate of decoherence is rapid. The result is a rapid decay, or relaxation, of the component of the magnetisation in the (x, y) plane.

The decoherence rate is effected by many factors, including:

- Changes in the effective Larmor frequency caused by local magnetic fields;
- Thermal excitations causing transient or long-term changes in the local magnetic field;
- Spin "mobility", i.e. the spin of one nucleus may be caused to flip due to the field perturbation introduced by the change of the spin state of a near-by nucleus. Such spin flips may be correlated such that the spin transition travels through the lattice;
- The presence of large molecules, paramagnetic ions or molecules; and
- Fields generated outside the tissue that may cause spin flips.

The mechanism by which the spin-spin interaction causes decoherence is separate from that by which M_z recovers. As a result, the spin-spin relaxation is independent of the spin-lattice relaxation. The relaxation of the transverse magnetisation, M_{xy}, is observed to satisfy:

$$\frac{dM_{xy}}{dt} = -\frac{M_{xy}}{T_2} \, ;$$

where T_2 is the time constant that characterises the spin-spin relaxation process. The decay of M_{xy} is given by:

$$M_{xy}(t) = M_0 \exp\left(-\frac{t}{T_2}\right) \, .$$

The spin-spin relaxation process is the result of the magnetic interaction of the nuclear spins with the spins of neighbouring nuclei and with the local magnetic environment. These magnetic interactions cause the effective Larmor frequency to depend on position. The result is effectively to create a position-dependent randomisation of precessional modes, leading to efficient depolarisation in the transverse plane. The result is that the time constant T_2 is typically $\lesssim 100\,\mathrm{ms}$, i.e. T_2 is typically small when compared to T_1.

1.1.4 Including free induction decay in the Bloch equation

The Bloch equation governing the time-evolution of the net magnetisation may be updated to include the spin-lattice and spin-spin relaxations to give:

$$\frac{d\mathbf{M}}{dt} = \gamma\left(\mathbf{M} \times \mathbf{B}_0\right) - \frac{\mathbf{M}_{xy}}{T_2} + \frac{M_0 - M_z}{T_1}\hat{\mathbf{k}} \, .$$

The first term describes the torque produced by the main uniform magnetic field $\mathbf{B}_0$. The second term describes the evolution of the transverse magnetisation vector $\mathbf{M}_{xy}$ due to the spin-spin interaction with time constant T_2. The third term describes the evolution of the longitudinal magnetisation M_z due to the spin-lattice interaction with time constant T_1. M_0 is the magnitude of the net magnetisation at equilibrium.

T_2, the intrinsic spin-spin relaxation time is determined by non-reversible thermodynamic processes at the nuclear level. In practice, the spin-spin relaxation time constant is reduced by a number of factors. A significant contribution comes from inhomogeneities in the main field $\mathbf{B}_0$. Such inhomogeneities give rise to reversible thermodynamic processes. The associated relaxation of the transverse magnetisation is characterised by a time constant T_2'. T_2 and T_2' many be combined to give the effective spin-spin relaxation time constant, T_2^* given by:

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'} \, ;$$

where $T_2' < T_2$ and so $T_2^* < T_2$.

2 Creating an image

To create an image requires that the signals generated by neighbouring tissues can be distinguished, i.e. contrast between different tissue types needs to be generated. To do this requires that tissue-dependent, i.e. distinguishable, signals must be localised to a region of space that is small enough for desired features to be resolved. The spin-lattice and spin-spin interactions that produce free induction decay of the NMR signal are used to produce signals that depend on tissue type, i.e. to generate contrast. Manipulation of the magnetic field strength across the tissue is used to provide spatial localisation.

2.1 Generating contrast

The rate of decay of the intensity of the RF waves generated during the free-induction decay is determined by T_1 and T_2. The properties of the medium determine the strength of the spin-lattice and spin-spin interactions and so T_1 and T_2 depend on the tissue that generates the signal. Typical values for T_1 and T_2 for different tissue types are reported in table 1.

Table 1: Spin-lattice, T_1, and spin-spin, T_2, relaxation time constants characteristic of various types of tissue.
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Tissue Type	T1 (ms)	T2 (ms)
Adipose tissues	240-250	60-80
Whole blood (deoxygenated)	1350	50
Whole blood (oxygenated)	1350	200
Cerebrospinal fluid (similar to pure water)	4200 - 4500	2100-2300
Gray matter of cerebrum	920	100
White matter of cerebrum	780	90
Liver	490	40
Kidneys	650	60-75
Muscles	860-900	50

While the rate of change of the signal strength is determined by T_1 and T_2, its magnitude is determined by the strength of the net magnetisation, i.e. by the number of ^{1}H nuclei contributing to the net magnetisation. The magnitude of the signal therefore depends on the concentration of ^{1}H in a particular tissue. As a result, different tissue types can be distinguished by measuring the signal magnitude, T_1 and T_2.

2.1.1 Determination of the spin-lattice relaxation time constant

Consider a system in which a sample is placed within the principal, uniform magnetic field, $\mathbf{B}_0$, and allowed to reach equilibrium such that the net magnetisation, $\mathbf{M}$, is parallel to $\mathbf{B}_0$ and of magnitude M_0. An RF magnetic-

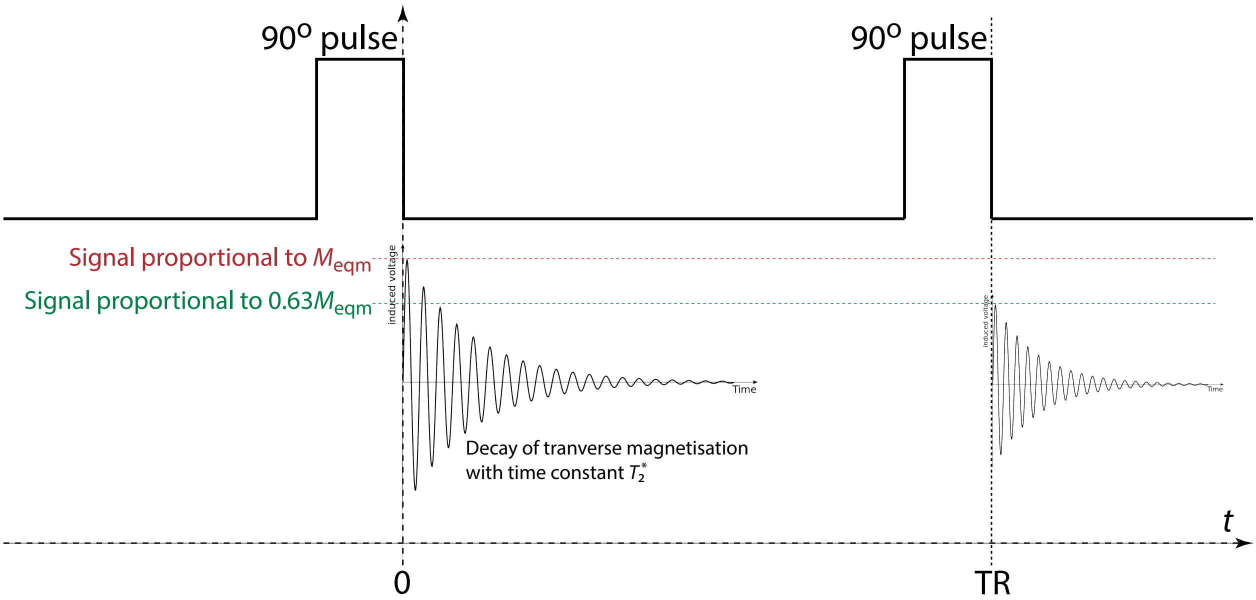

Figure 1: Schematic diagram of a segment of a pulse sequence used to determine T_1. The application of the RF $\mathbf{B}_1$ pulses are shown as the solid black lines. The oscillating grey lines show the RF signals generated during free induction decay. The time to repetition, TR, is indicated. The first 90° pulse is applied when the net magnetisation is in its equilibrium configuration, i.e. $\mathbf{M}$ is parallel to $\mathbf{B}_0$. At the end of this first pulse, at time $t = 0$, the signal generated from the precession of $\mathbf{M}$ is maximal. The rate of decay of the signal amplitude is governed by the time constant T_2^*. At the end second 90° pulse the signal amplitude is reduced.

field pulse, $\mathbf{B}_1$, is applied for a time, t_{90}, long enough for the net magnetisation to rotate through 90° so that, when the RF pulse ends, the net magnetisation is precessing in the (x, y) plane.

Take $t = 0$ to be the time at which the 90° pulse ends. Then, the magnitude of transverse magnetisation, M_{xy}, at $t = 0$ is given by:

$$M_{xy}(t = 0) = M_{xy}(0) = M_0 \, .$$

Since the magnetisation is now precessing in the (x, y) plane, the longitudinal magnetisation, M_z, is zero. M_{xy} decays exponentially, in accordance with the Bloch equation.

The Bloch equation implies that the longitudinal magnetisation, M_Z, recovers according to:

$$M_z(t) = M_0 \left[1 - \exp\left(-\frac{t}{T_1}\right)\right] \, .$$

So, for $t \gtrsim 5T_1$, $M_z - M_0 \lesssim 0.5\%$, i.e. the longitudinal magnetisation has recovered. Since T_2^* is usually much less than T_1, the transverse magnetisation has decayed to zero significantly before the longitudinal magnetisation has recovered. Therefore, if a second 90° pulse is applied for $t < 5T_1$, the resulting M_{xy} will be less than M_0. The evolution of the signal is shown schematically in figure 1. For example, if the second 90° pulse is applied at $t = T_1$, then $M_{xy}(t = T_1) = 0.63M_0$. The time between the first and second 90° pulse is referred to as the “time to repetition”, TR.

At TR, the longitudinal polarisation has recovered to yield:

$$M_z(\mathrm{TR}) = M_0 \left[1 - \exp\left(-\frac{\mathrm{TR}}{T_1}\right)\right] \, . \tag{1}$$

So, the repetition of the 90° pulse at $t = \mathrm{TR}$ gives $M_{xy}(\mathrm{TR}) = M_z(\mathrm{TR})$ and therefore T_1 can be extracted by measuring $M_{xy}(\mathrm{TR})$ as a function of TR. The pulse sequence described here is referred to as a “partial saturation pulse sequence” since the longitudinal polarisation has not been allowed to recover fully.

An alternative procedure by which to measure T_1, is to invert the net magnetisation by using an RF $\mathbf{B}_1$ pulse twice as long as that required to rotate the net magnetisation by $90°$, i.e. $t_{180} = 2t_{90}$. If, at the start of the "180°" pulse, the net magnetisation is in its equilibrium state parallel to $\mathbf{B}_0$, then it will be anti-parallel to $\mathbf{B}_0$ at the end of the "180°" pulse. As before, $t = 0$ may be taken to be the instant at which the "180°" pulse ends. The evolution of the longitudinal magnetisation is now given by:

$$M_z(\mathrm{TI}) = M_0 \left[1 - 2\exp\left(-\frac{\mathrm{TI}}{T_1}\right)\right] ;$$

where TI is the "time from inversion". The application of a $90°$ pulse at TI will sample the longitudinal magnetisation at TI. In the same way as before, $M_{xy}(\mathrm{TI}) = M_z(\mathrm{TI})$ and therefore T_1 can be extracted by measuring $M_{xy}(\mathrm{TI})$ as a function of TI. The pulse sequence and evolution of the signal is shown schematically in figure 2

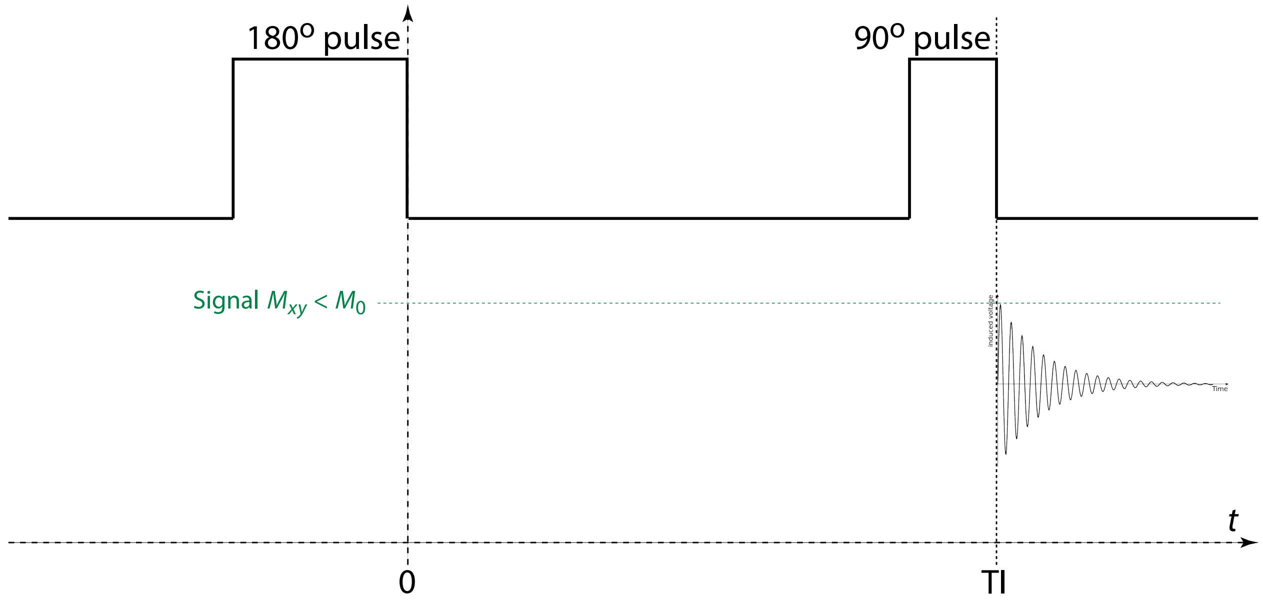

Figure 2: Schematic diagram of a segment of a pulse sequence from an inversion recovery pulse sequence used to determine T_1. The application of the RF $\mathbf{B}_1$ pulses are shown as the solid black lines. The oscillating grey lines show the RF signals generated during free induction decay. The time from inversion, TI, is indicated.

2.1.2 Determination of the spin-spin relaxation time constant

Once again, consider a system in which a sample is placed within the principal, uniform magnetic field, $\mathbf{B}_0$, and allowed to reach equilibrium such that the net magnetisation, $\mathbf{M}$, is parallel to $\mathbf{B}_0$ and of magnitude M_0. A $90°$ degree pulse is applied to rotate the the net magnetisation into the (x, y) plane. As before, $t = 0$ is taken to be the instant at which the $90°$ pulse ends. We now focus on the evolution of the transverse component of the magnetisation, $\mathbf{M}_{xy}$.

At the microscopic level, the rate of precession of the individual ^{1}H spins is determined by the local magnetic environment. Some ^{1}H nuclei precess faster than the nominal Larmor frequency, some precess more slowly. This results in a decoherence of the precessions of the ^{1}H nuclei that were initially coherent at $t = 0$, i.e. when the RF $\mathbf{B}_1$ pulse ended. The rate of decay of the transverse magnetisation is T_2^*.

Recall that the mechanism by which the transverse magnetisation decays is that the phase coherence in the precession of the individual ^{1}H nuclei is gradually destroyed by local variations in the magnetic field strength. Lets define "synchronous nuclei" to be those nuclei that experience a magnetic field strength $\mathbf{B}_0$ and therefore

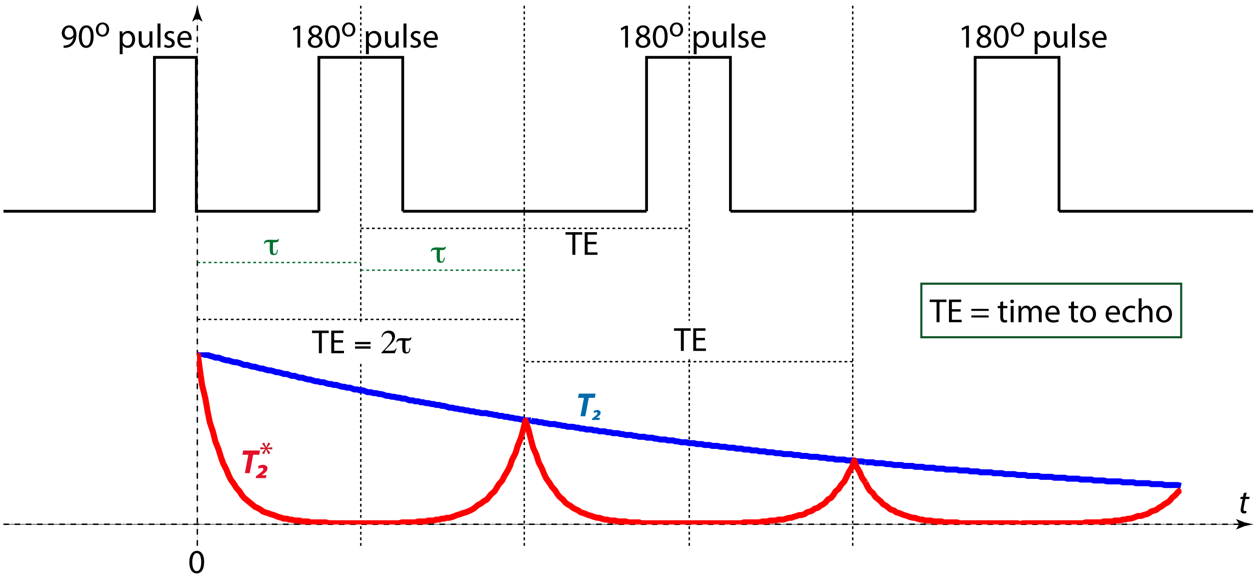

Figure 3: Schematic diagram of a segment of a spin-echo pulse sequence used to determine T_2. The application of the RF $\mathbf{B}_1$ pulses are shown as the solid black lines. The oscillating grey lines show the RF signals generated during free induction decay. The solid red line indicates the decay of M_{xy} with time constant T_2^* which determines the amplitude of the RF signal which also decays with time constant T_2^*. The solid blue line indicates the decay in M_{xy} with time constant T_2. The time to echo, TE, is indicated.

rotate at the Larmor frequency ω_0. Nuclei that are in a location where the field strength is relatively stronger than $\mathbf{B}_0$ precess with angular velocity greater than ω_0. Conversely, nuclei that experience a field strength is relatively weaker than than induced by $\mathbf{B}_0$ precess with angular velocity smaller than ω_0. After a time $t = \tau$, the nuclei that are precessing faster than ω_0 will have a phase that is more advanced than the synchronous nuclei while those that are precessing at a rate slower than ω_0 will have a phase that lags behind that of the synchronous nuclei.

Consider the effect of a 180° pulse ends applied to the system at time $t = \tau$. At the end of the 180°, the magnetisation vectors of all ^{1}H nuclei will be inverted. The result of this inversion will be to take $M_z \to -M_z$ and to add 180° degrees to the phase location of each nucleus in its precession in the (x, y) plane. The impact of the phase inversion on those nuclei that had phases that were behind the synchronous nuclei, i.e., that were precessing at a rate slower than ω_0, being flipped such that their phase now in advance of the synchronous nuclei.

The reversal of the phase position of the nuclei in the (x, y) plane is not random, those that were precessing fastest now lag in phase the most, while those that were precessing most slowly are now most advanced in phase. As a result, after a further delay of τ, i.e. at $t = 2\tau$, the spins have recovered the relative phases they had at the end of the initial 90° pulse, with the result that the RF signal produced by the precession of the net magnetisation is an echo of that produced at $t = 0$. This process is shown schematically in figure 3. The time $t = 2\tau$ is referred to as the "time to echo", i.e. $\text{TE} = 2\tau$. The magnitude of M_{xy}, and therefore the amplitude of the RF signal, at $t = 2\tau$ is less than that at $t = 0$ by an amount determined by the spin lattice time constant T_2, i.e.:

$$M_{xy}(\text{TE}) = M_0 \exp\left(-\frac{\text{TE}}{T_2}\right) .$$

T_2 can be extracted by measuring $M_{xy}(\text{TE})$ as a function of TE.