# J-PARC Superconducting Combined-Function Magnet System Model Upgrade

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# Motivation

- Main goal of the study is to improve the modelling of the Neutrino Primary Beamline:
    - to reduce discrepancies between measurements and model simulations
    - to reduce beam loss
    - to facilitate beam tuning

- A first step was taken by Jaroslaw by modelling the primary beam line using the BeamOptics (BO) code
    - significant differences between BO and SAD models were identified

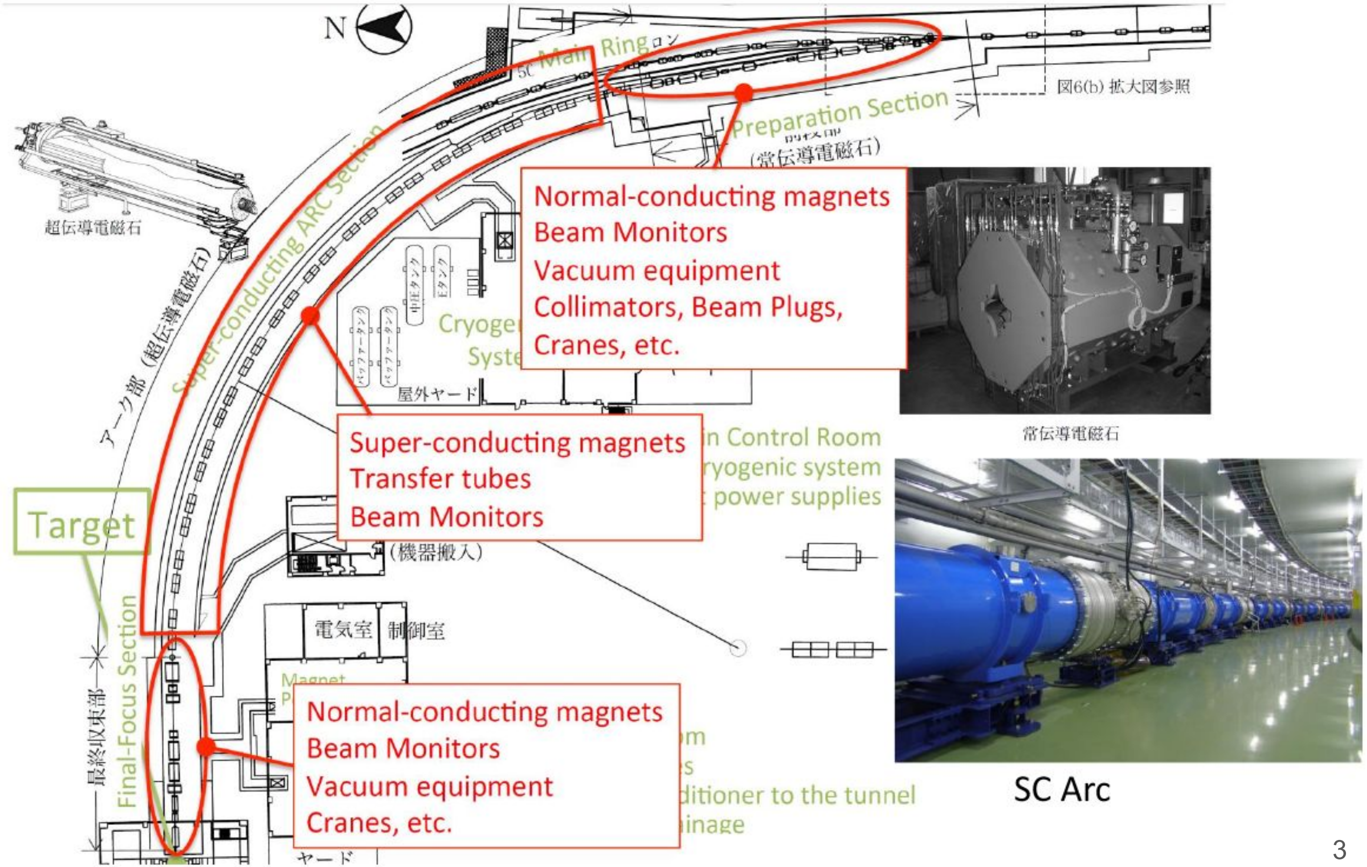
# Neutrino Primary Beamline

- Stably delivers protons from the Main Ring to the neutrino production target

- Preparation section

- Arc section

- Final-Focus section



# Arc Section

- bends the proton beam by 80.7 degrees towards the SK detector
- contains 14 identical doublets of superconducting combined-function magnets
- each doublet contains one defocusing (D) and one focusing (F) quadrupole fields
- currently the magnetic fields are modelled using the *hard-edge approximation*

# Fringe field

- Included the fringe field in the model (soft-edges)
- Fringe field described analytically by:

$$F = \frac{1}{1+exp(P(s))}$$

where s is the distance to the effective field boundary (EFB) and

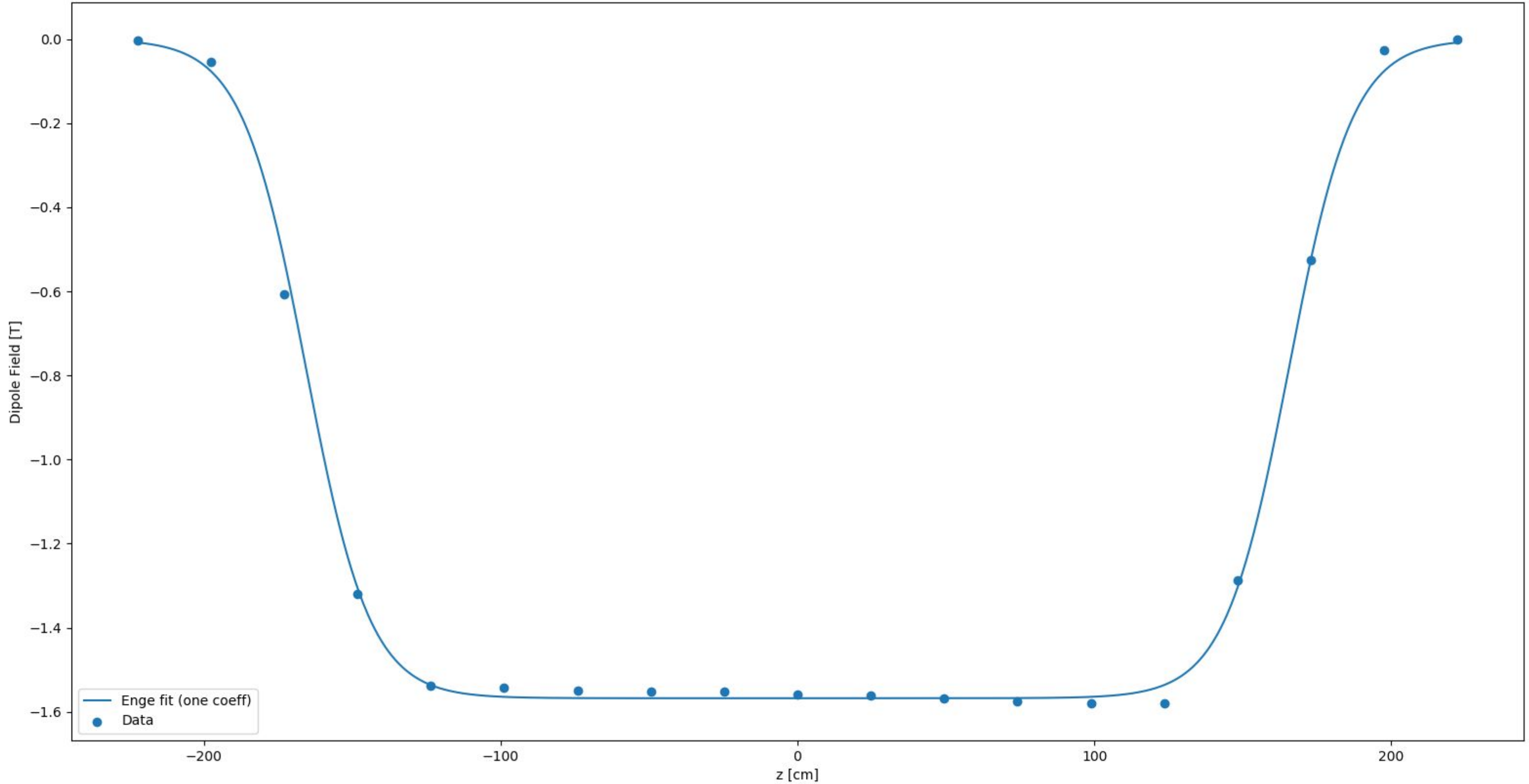
$$P(s) = C_0 + C_1\left(\frac{s}{\lambda}\right) + C_2\left(\frac{s}{\lambda}\right)^2 + C_3\left(\frac{s}{\lambda}\right)^3 + C_4\left(\frac{s}{\lambda}\right)^4 + C_5\left(\frac{s}{\lambda}\right)^5$$

where λ is the fringe field extent and C0-C5 are known as Enge coefficients

- Started with the basic case where the field is assumed to be symmetric and all coefficients apart from C1 are zero

- In this simple model, the following function is fitted to the dipole field data:

$$F = \frac{B_0}{2}(tanh((\frac{C_1}{\lambda})(\frac{L}{2} - z)) + tanh((\frac{C_1}{\lambda})(\frac{L}{2} + z)))$$



# Cell model

- cell is modelled in PyZgoubi
- used geometry parameters from the description shown

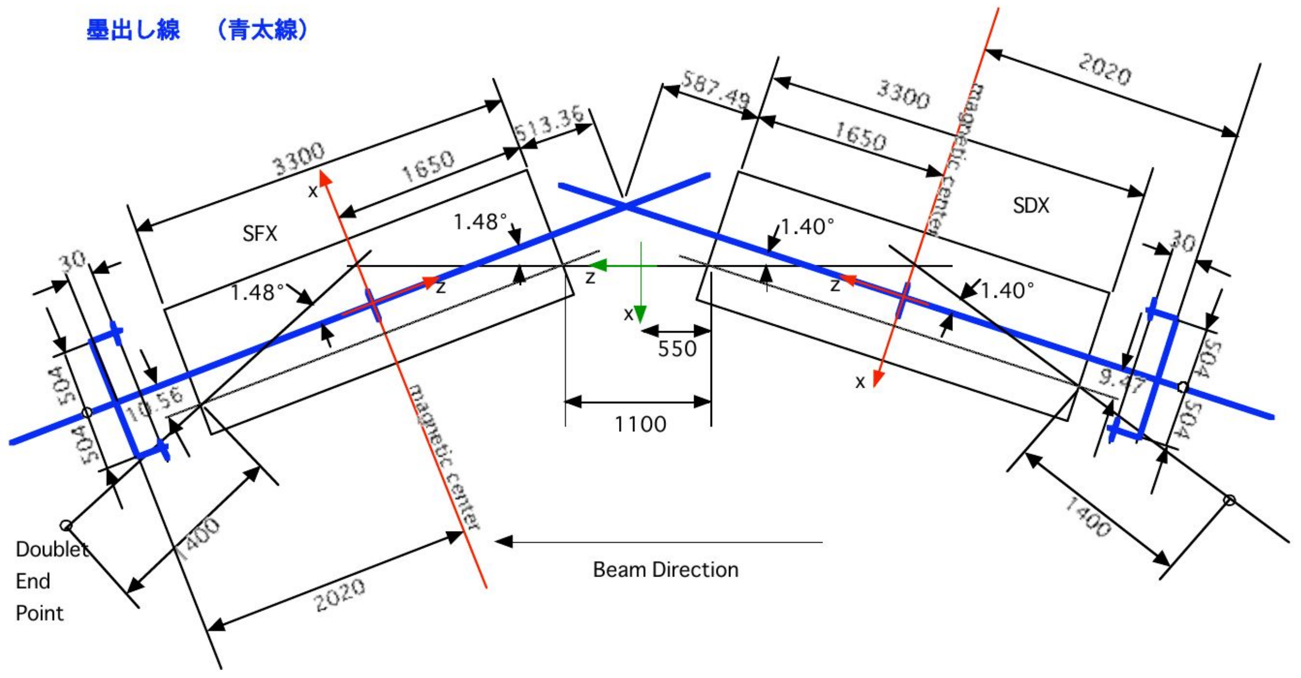
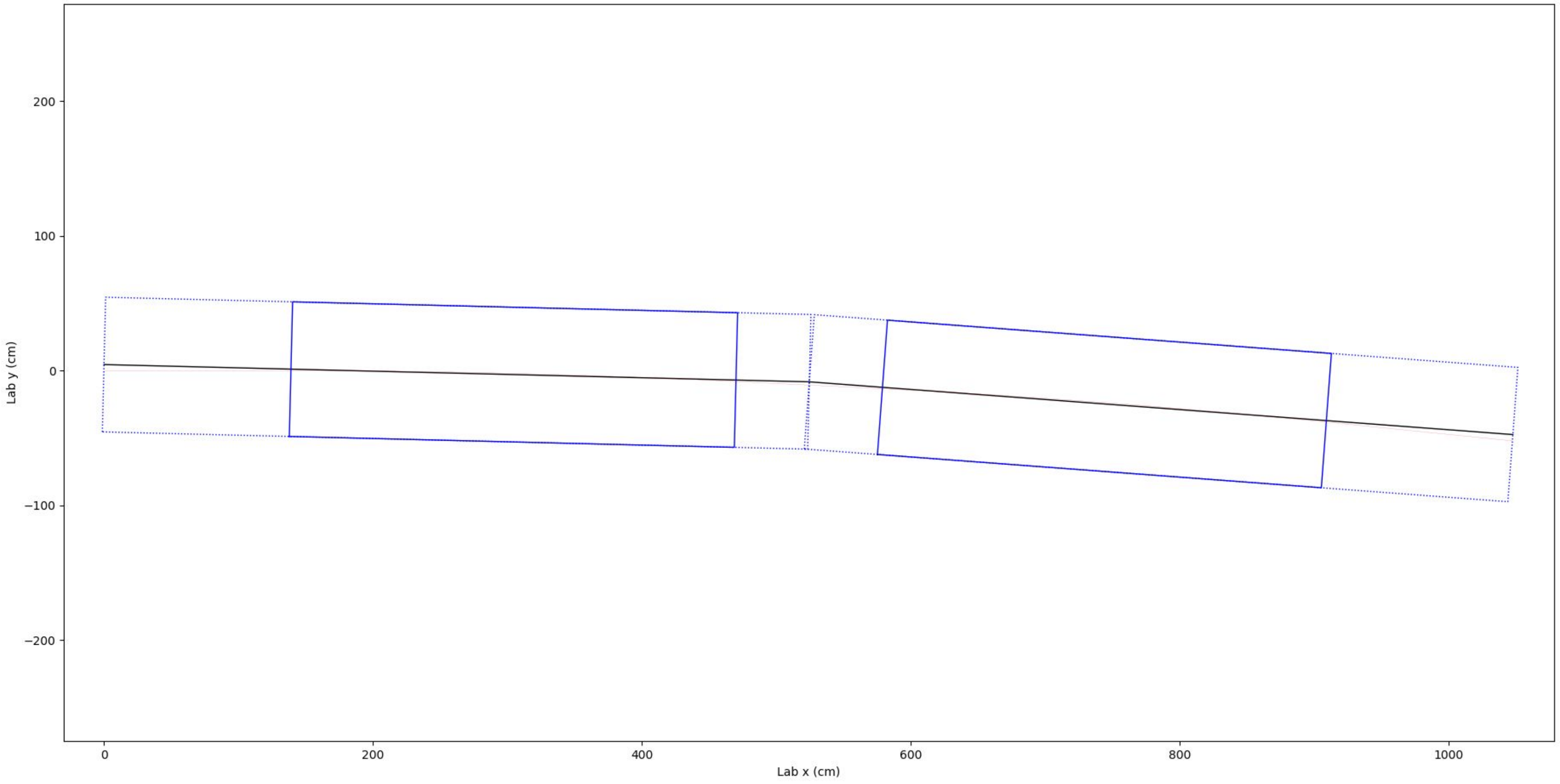
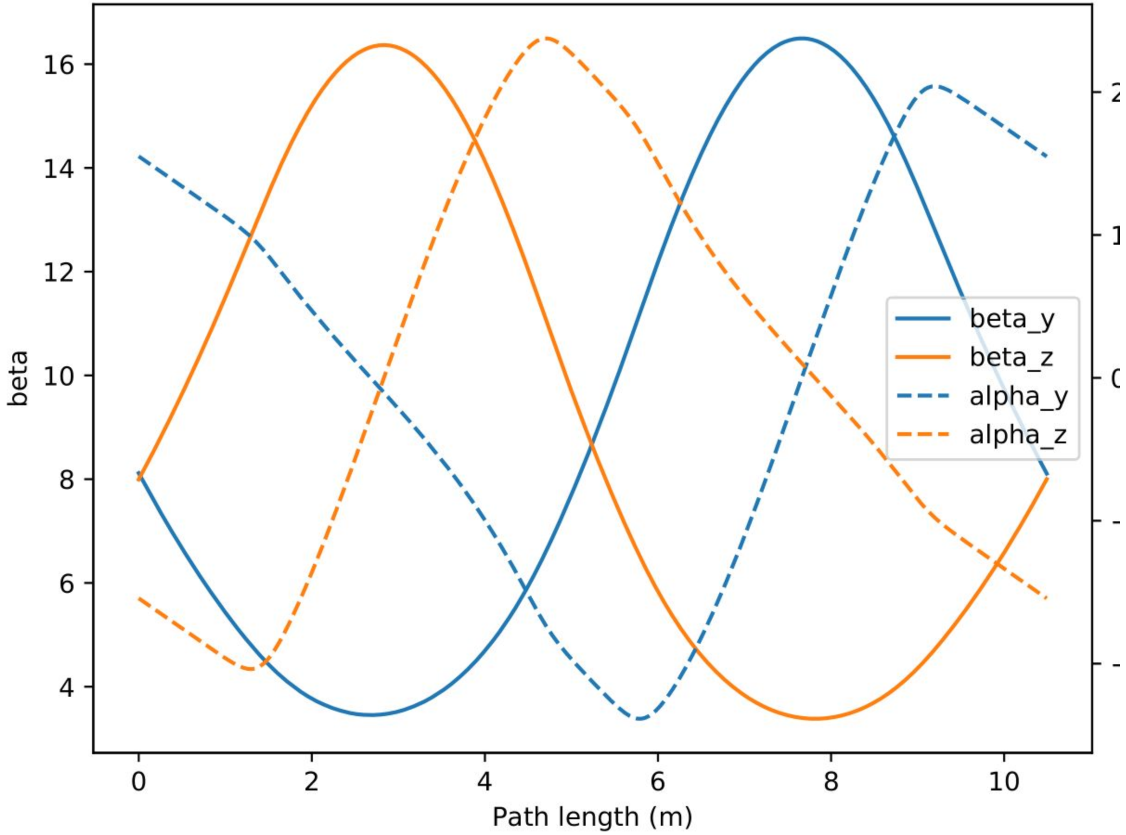



図４　J-PARC ニュートリノ超伝導磁石クライオスタットの粗設置用の墨出し線
上から見た図。Y 軸は上に向かう。

# Cell model - PyZgoubi



# Cell Twiss profiles and tune




Q(h, v) = (0.24287, 0.24678)

Horizontal transfer matrix

$$\begin{matrix} 1.59217 & 8.09814 & 0.34495 \\ -0.41900 & -1.50254 & 0.02402 \\ 0 & 0 & 1 \end{matrix}$$

Vertical transfer matrix

$$\begin{matrix} -1.52237 & 7.99561 & 0 \\ -0.42263 & 1.56282 & 0. \\ 0 & 0 & 1 \end{matrix}$$

# Comparison with Jaroslaw's hard edge model in Beam Optics

Transfer matrix residuals

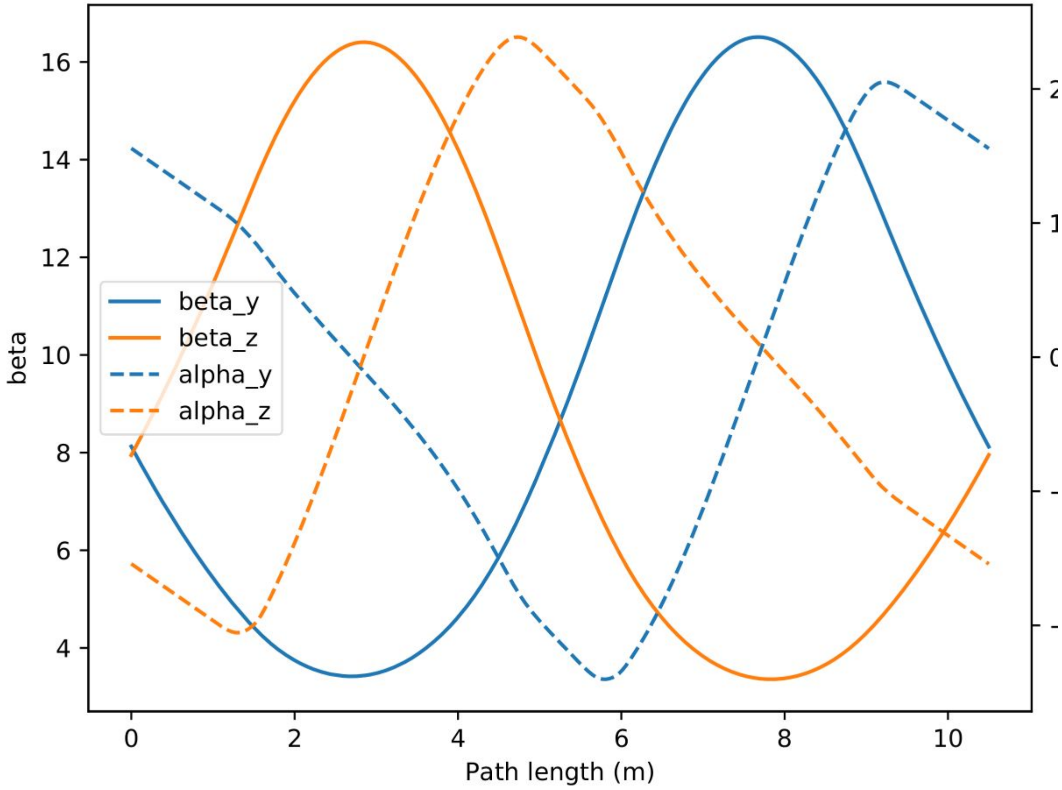
Horizontal

$$\begin{matrix} 0.02578 & 0.0586 & 0.00254 \\ -0.00087 & 0.0051 & -0.00074 \\ 0 & 0 & 0 \end{matrix}$$

Vertical

$$\begin{matrix} -0.0105 & -0.02020 & 0 \\ -0.00392 & 0.00428 & 0 \\ 0 & 0 & 0 \end{matrix}$$

# 6 Enge coefficients: Cell Twiss profiles and tune




Q(h, v) = (0.24433, 0.24778)

Horizontal transfer matrix

$$\begin{pmatrix} 1.59142 & 8.11508 & 0.34517 \\ -0.42147 & -1.52007 & 0.02304 \\ 0 & 0 & 1 \end{pmatrix}$$

Vertical transfer matrix

$$\begin{pmatrix} -1.52744 & 7.94903 & 0 \\ -0.424677 & 1.55539 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

# Transfer matrix from beam measurements

- along the primary beamline, Segmented Secondary Emission Monitors (SSEM) are used to measure the beam profile
- beam center and width information is available at four different positions within the arc section (SSEM10, 11, 12, 13)
- currently working on transfer matrix calculation using the SSEM data
- transfer matrix parameterised using Twiss functions and phase advance per cell

$$\begin{pmatrix} cos(\phi) + \alpha sin(\phi) & \beta sin(\phi) \\ -\gamma sin(\phi) & cos(\phi) - \alpha sin(\phi) \end{pmatrix}$$

# Beam width

Use beam width to calculate the beam betatron function at the SSEM positions, assuming constant emittance.

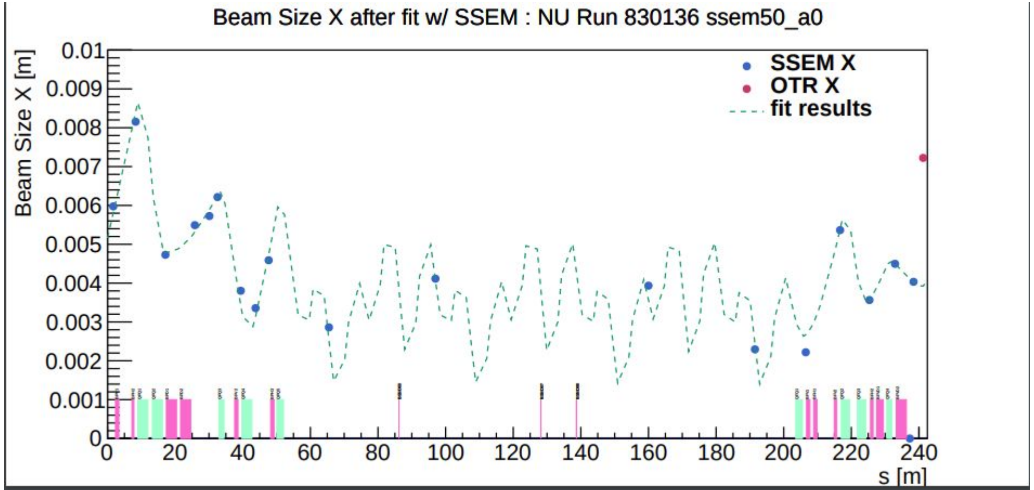
$$\beta = \frac{\sigma^2}{\epsilon}$$

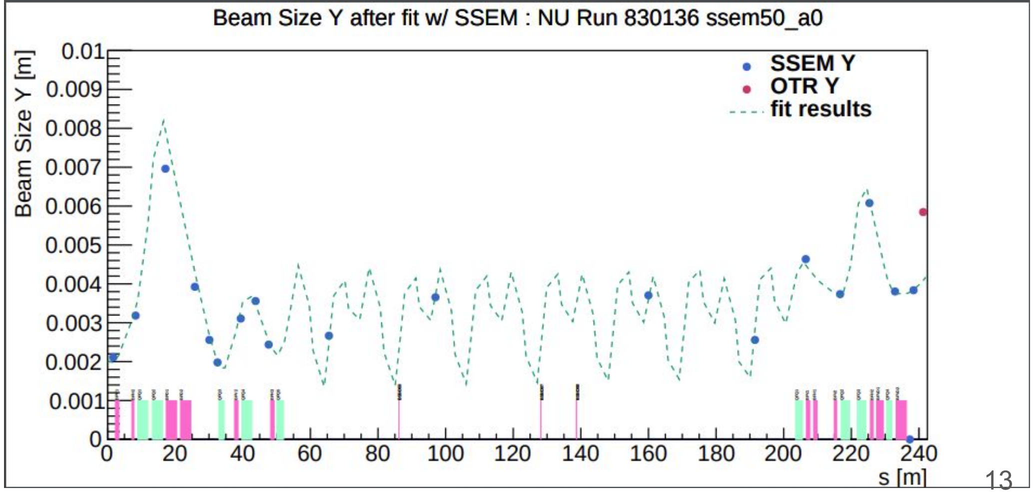
Then use the Twiss parameters transformation to fit/calculate for the cell transfer matrix parameters:

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix} = \begin{pmatrix} C^2 & -2SC & S^2 \\ -CC' & SC' + S'C & -SS' \\ C'2 & -2S'C' & S'^2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix}$$

In particular, make use of the top row equation:

$$\beta_f = C^2 \beta_0 - 2SC\alpha_0 + S^2 \gamma_0$$



# Beam width

Multiple cells between the SSEMs. Can parameterise further:

$$
\begin{aligned}
C_n &= cos(n\phi) + \alpha sin(n\phi) \\
S_n &= \beta sin(n\phi)
\end{aligned}
$$

where n is the number of cells and $\alpha$, $\beta$, $\phi$ are the one-cell transfer matrix parameters. Now the beta transport equation reads,

$$\beta_f = C_n^2 \beta_0 - 2 S_n C_n \alpha_0 + S_n^2 \frac{1+\alpha_0^2}{\beta_0}$$

One equation, 4 unknowns ($\alpha$, $\beta$, $\phi$, $\alpha_0$). Can use multiple SSEM measurements.

# Beam width

Using transport equations from SSEM 10->11, 10->12, 10->13

$$\beta_{11} = C_3^2\beta_{10} - 2S_3C_3\alpha_{10} + S_3^2\frac{1+\alpha_{10}^2}{\beta_{10}}$$
$$\beta_{12} = C_9^2\beta_{10} - 2S_9C_9\alpha_{10} + S_9^2\frac{1+\alpha_{10}^2}{\beta_{10}}$$
$$\beta_{13} = C_{12}^2\beta_{10} - 2S_{12}C_{12}\alpha_{10} + S_{12}^2\frac{1+\alpha_{10}^2}{\beta_{10}}$$

One can solve for $\beta$ and $\phi$, while $\alpha$ is dependent on $\alpha_0$. The phase advance reads:

$$\phi = \frac{1}{3}[arctan(\pm\sqrt{\frac{-\beta_{10}-2\beta_{11}+2\beta_{12}+\beta_{13}}{-4\beta_{11}+4\beta_{12}}}, \pm\sqrt{\frac{-\beta_{10}+2\beta_{11}-2\beta_{12}+\beta_{13}}{4\beta_{11}-4\beta_{12}}}) + 2\pi n]$$

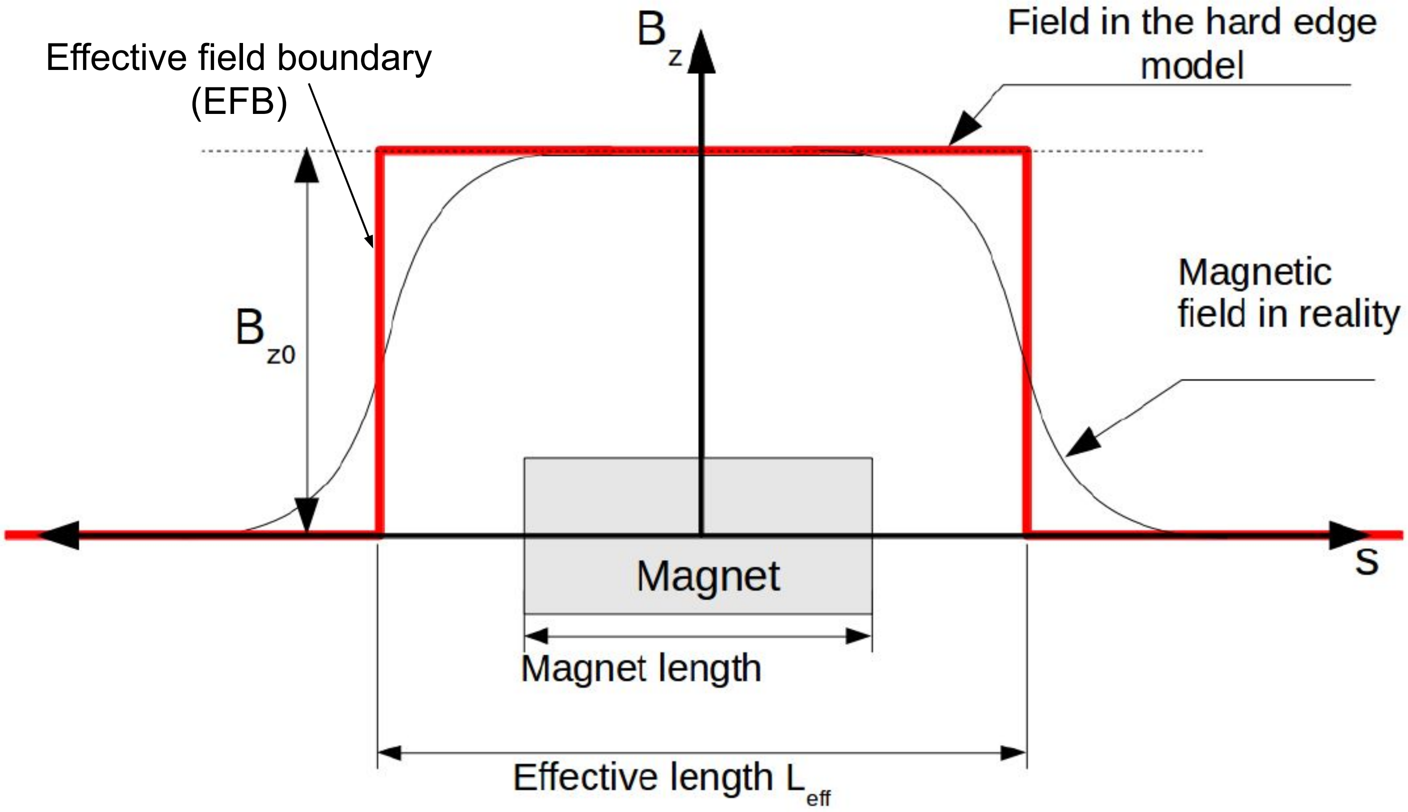
$arctan(x, y) = arctan(\frac{y}{x})$, taking into account the quadrant of (x,y)

# Further work

- Finalise calculation of cell transfer matrix using beam data and compare with the updated model and old model
- Expand model to the entire arc section
- Possible involvement in the study of beam optics for a 20-25 GeV proton beam

# Back-up

# Hard-edge approximation vs real field



# Magnet parameters

**D magnet**

B1 = -1.5595 T

L = 3010.0 mm

C1 = 1.6099

$\lambda$ = 343.837 mm

QGrad = -11.2110 T/m

**F magnet**

B1 = -1.5644 T

L = 3010.0 mm

C1 = 1.6059

$\lambda$ = 343.881 mm

QGrad = 11.2193 T/m

# Twiss parameters

- beam shape in phase-space defined by an ellipse
- use 4 parameters to characterise the ellipse area, shape and orientation
- tune (of a lattice cell) - the number of revolutions in the phase-space a particle undergoes while passing once through the cell