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Narrow beams and the Bragg peak

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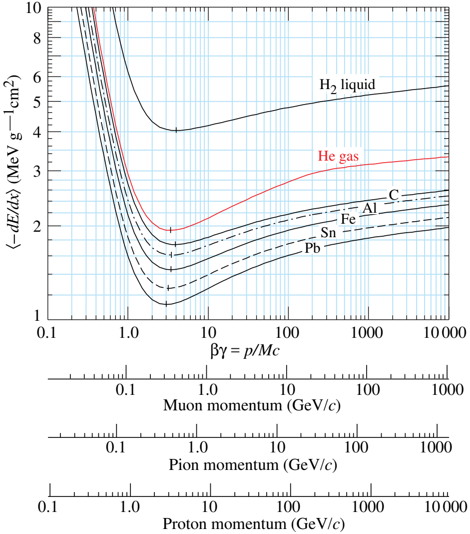
Ionisation energy loss



Ionisation energy loss:

$$\left\langle \frac{dE}{dx} \right\rangle = Kz^2 \frac{Z}{A} \left[\frac{1}{2} \ln \left(\frac{2m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} \right) - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

Bortfeld parameterisation:

Energy fluence, Ψ:

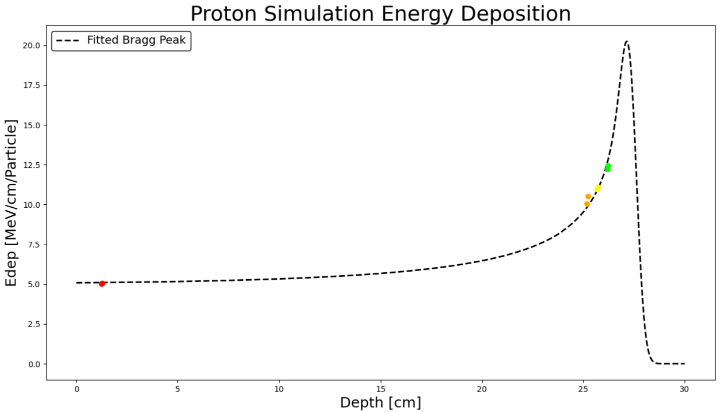
$$\Psi = \Phi(x) E_{\mathrm{remain}}(x)$$

"Terma", T:

$$T(x) = -\frac{1}{\rho} \frac{d\Psi}{dx}$$

Terma is "Differential dose deposited" over fixed area

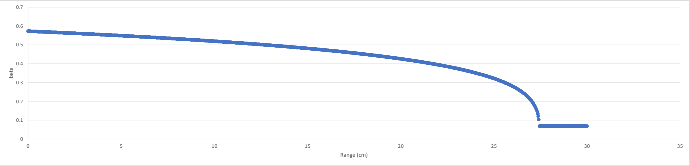
Bortfeld parameterisation



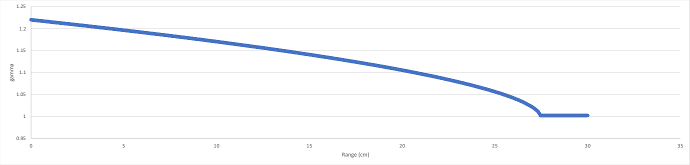
HT's implementation of Bortfeld parameterisation for 206 MeV kinetic energy protons in water

Proton slowing down

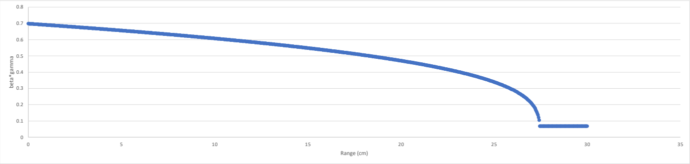
β versus range (cm)



γ versus range (cm)

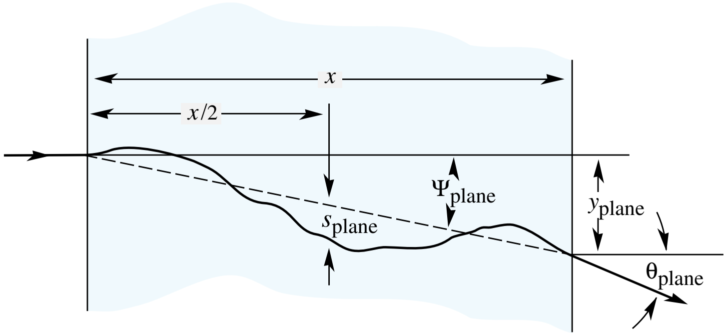


$\beta\gamma$ versus range (cm)



Multiple Coulomb scattering

$$y^{\rm rms}_{\rm plane} = \frac{1}{\sqrt{3}} x \frac{13.6\,\mathrm{MeV}}{\beta c p} z \sqrt{\frac{x}{X_0}} \left[1 + 0.038 \ln \left(\frac{x^2 z^2}{X_0 \beta^2} \right) \right] \propto \frac{1}{\beta^2}$$



Calculation for a prototypical beam

- Take Terma, T, from HT's calculation:

$$T(x) = -\frac{1}{\rho}\frac{d\Psi}{dx}$$

- Area illuminated by beam grows due to MCS
- Take radius, r, of the beam at $x = 0$ to be $r_0 = 0.5\,\text{cm}$
- At $x > 0$, radius of beam given by: $r \approx r_0 + y_{\text{plane}}$
    - Approximate because y_{plane} is an "RMS" quantity
- So instantaneous dose deposited in a step Δx is:

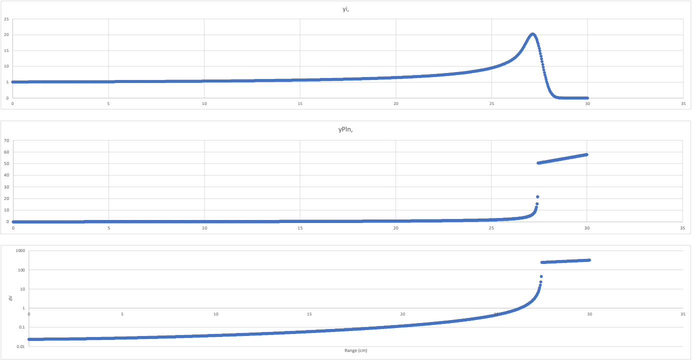
$$\Delta D = \frac{1}{\pi r^2 \Delta x}(T\Delta x) \approx \frac{1}{\pi(r_0 + y_{\text{plane}})^2} T$$

Elements of calculation of dose

T versus range (cm)

$\mathbf{y}_{\mathrm{plane}}$ versus range (cm)

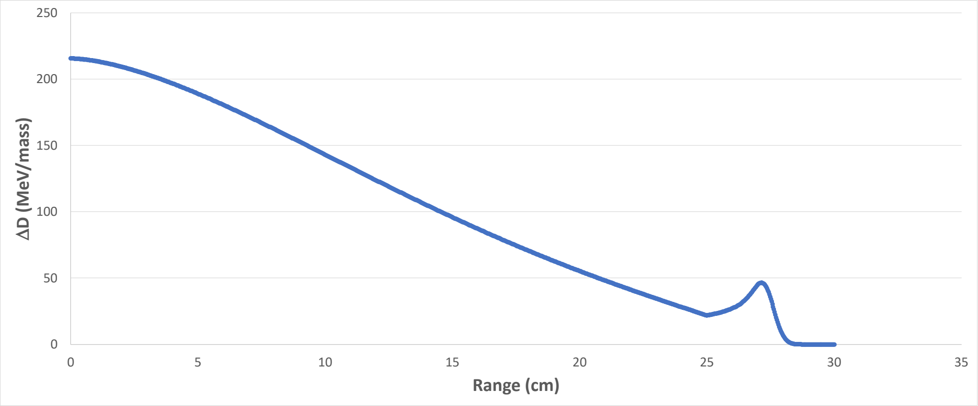
ΔV versus range (cm)



For next slide: arbitrarily assume MCS dominated growth of beam width "stops" at $x = 25\,\mathrm{cm}$

ΔV calculated for a beam that initially has a radius $r_0 = 0.5\,\mathrm{cm}$

Evolution of instantaneous dose, D



Exact shape depends on r_0, "regularisation" of MCS ...

Comments

- Energy loss, divergence of beam, much more complicated that "just" electromagnetic effects
- Need to be sure to understand various energy loss and scattering mechanisms in more detail
- Does Geant4 get it right? Some indication from HT's fits that it does, but, need to be quantitative
- Need to take some care with energy density and dose
- What are the consequences for the acoustic signal?