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Nuclear medicine

Week 4; Lecture 4; Section 2: SPECT: reconstruction

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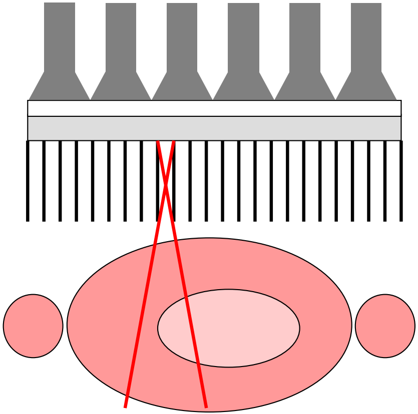
Section 2

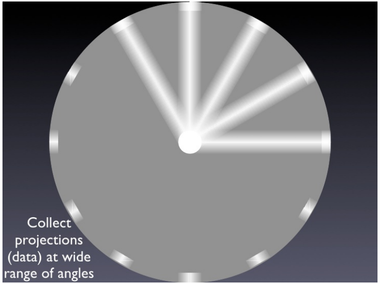
Reconstruction

Projection on image plane

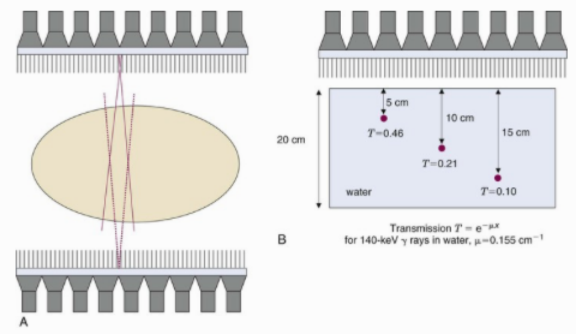


Absorptive collimator means that each hole views a pencil-like area of the object

Repeat for a wide range of angles:



Geometric response; conjugate counting



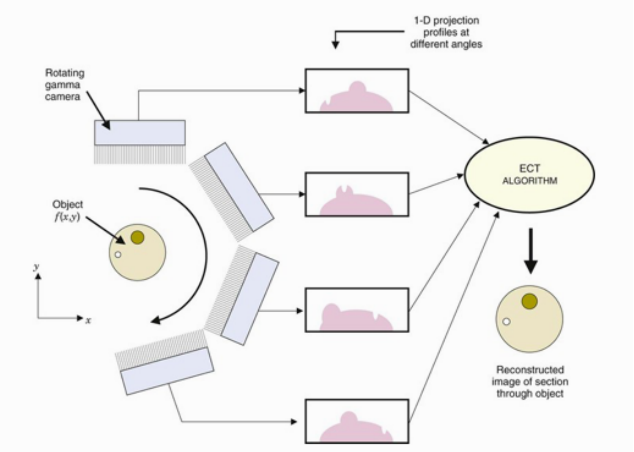
$I(r, \phi) \neq I(r, \phi + \pi)$

- Attenuation of γ intensity
- Divergence of image cone
- Measurements at ϕ and $\phi + \pi$ "conjugate"

Example of attenuation of γs from ^{99m}Tc:

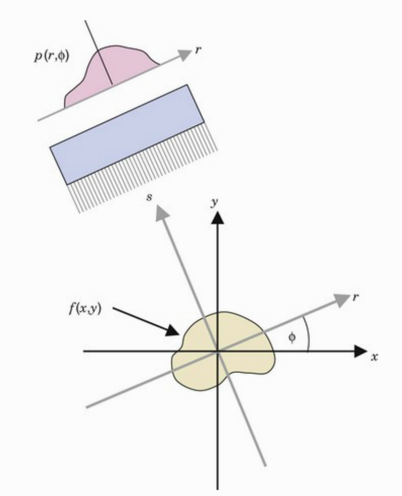
- Exploit conjugate measurements to correct for lost attenuation

Back projection



ECT: Emission computed tomography
TCT: Transmission computed tomography

Back projection: local coordinate system



Local coordinate system r, s:

- r: coordinate along gamma camera
- s: distance camera to source

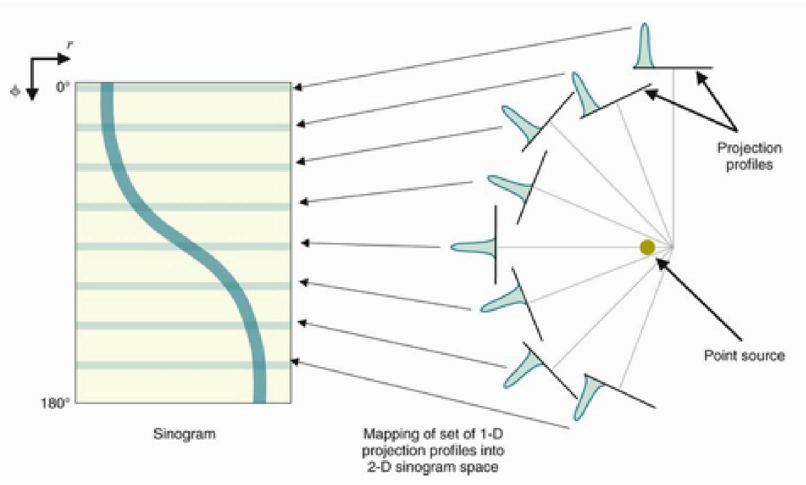
r, s coordinates related to x, y by:

$$\begin{aligned} r &= x\cos\phi + y\sin\phi \\ s &= -x\sin\phi + y\cos\phi \end{aligned}$$

x, y may be reconstructed using:

$$\begin{aligned} x &= r\cos\phi - s\sin\phi \\ y &= r\sin\phi + s\cos\phi \end{aligned}$$

Back projection: sinogram



Back projection: profile construction

Measurement at each projection measures a response “profile”, $p(r, \phi)$

Want to reconstruct the activity in a particular slice, $f(r, s)$ or $f(x, y)$

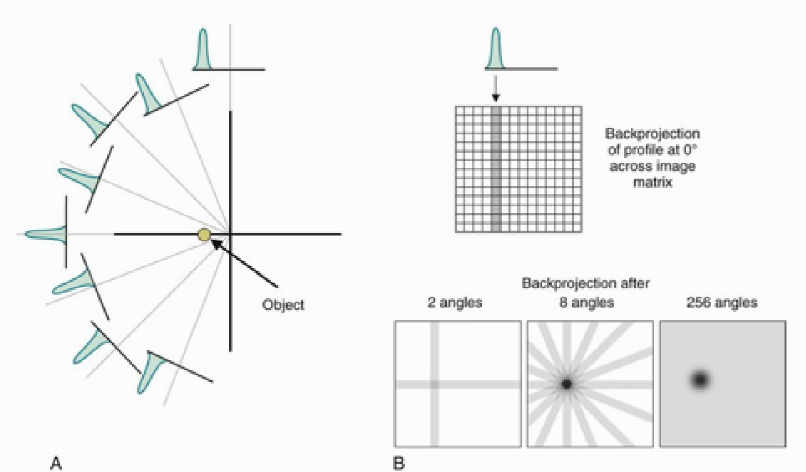
In “simple” back projection, the total response measured at a particular r_i, ϕ_i is divided between the pixels along the projected coordinate s

The total (uncorrected) activity, $f'(x, y)$ within a pixel at coordinate x, y is then given by:

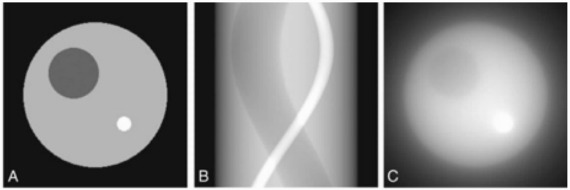
$$f'(x, y) = \frac{1}{N} \sum_{i=1}^{N} p(r_i, \phi_i)$$

where the sum runs over the N projections that illuminate the pixel at x, y

Back projection: illustration



Back projection: illustration using a simple phantom



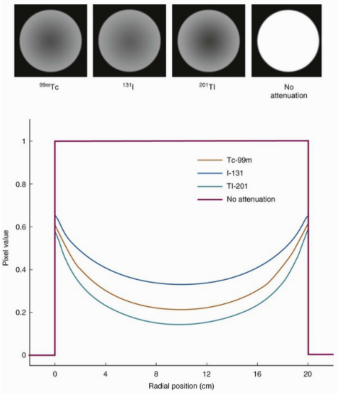
Main features of phantom appear in the image . . . but . . .

The attribution of activity to pixels along the projected coordinate s causes “spoke-like” image with few projections

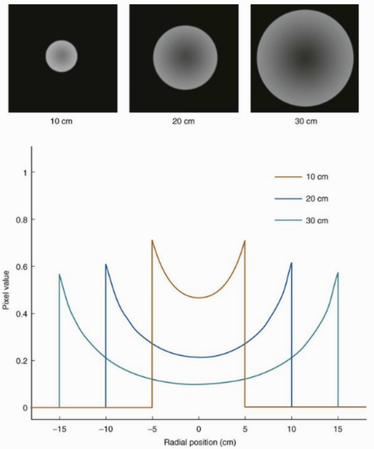
More projections improve image, but, attribution of activity leads to apparent activity outside the object and blurring of the image

More sophisticated reconstruction algorithms (e.g. filtered back projection, see later) have been developed to overcome this defect

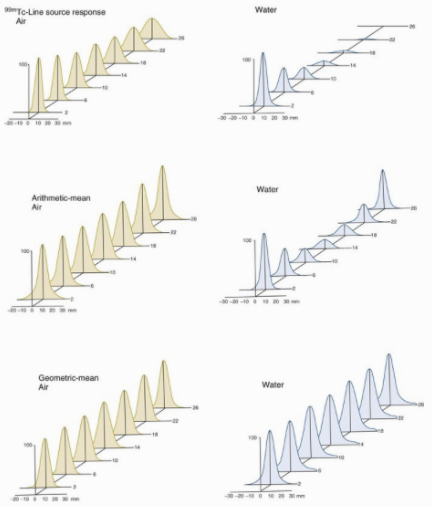
Attenuation depends on γ energy and depth



$$E_\gamma^{99m\mathrm{Tc}} = 140\,\mathrm{keV};\ E_\gamma^{131\mathrm{I}} = 364\,\mathrm{keV};$$
$$E_\gamma^{201\mathrm{Tl}} = 70\,\mathrm{keV}$$



Attenuation

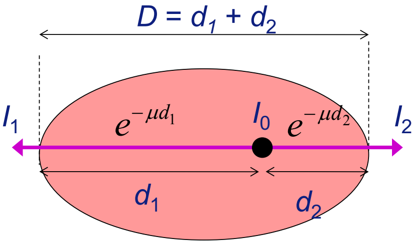


Example:

- High-resolution pin-hole collimator
- Resolution for line source diameter 2.5 mm:
  - As a function of distance source$\rightarrow$detector
  - In air (left) and in water (right)
- Corrections applied:
  - Top: no correction
  - Middle: arithmetic mean:
    $$I_A = \frac{1}{2}(I_1 + I_2)$$
  - Bottom: geometric mean:
    $$I_G = (I_1 \times I_2)^{\frac{1}{2}}$$

Geometric mean gives most uniform response

Geometric mean



Intensity measured in two conjugate PMTs, numbered 1 and 2:

$$
\begin{aligned}
I_1 &= I_{01}\exp(-\mu d_1) \\
I_2 &= I_{02}\exp(-\mu d_2)
\end{aligned}
$$

Geometric mean; I_G:

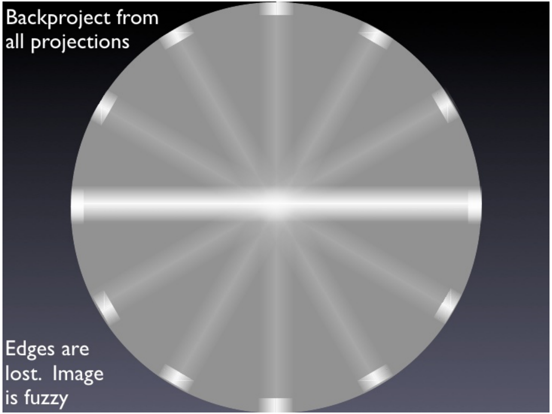
$$
\begin{aligned}
I_G^2 &= I_{01} \times I_{02} \\
&= I_{01} \times I_{02} \exp\left(-\mu(d_1 + d_2)\right)
\end{aligned}
$$

If $I_0 = I_{01} + I_{02}$:

$$
I_G = I_0 \exp\left(-\mu \frac{D}{2}\right)
$$

I.e. I_G depends on total depth D rather than d_1 or d_2.
The result is exact only for homogeneous media and point sources. Corrections can be derived to accommodate these effects.

Attenuation correction



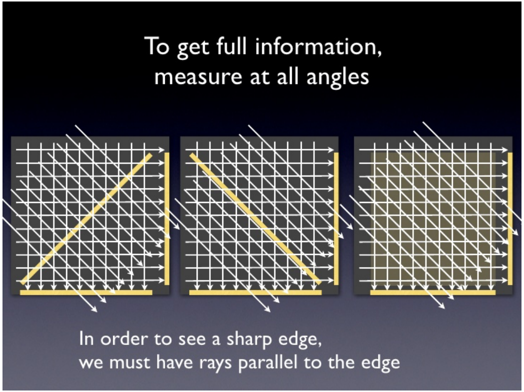
Define, attenuation correction factor, ACF:

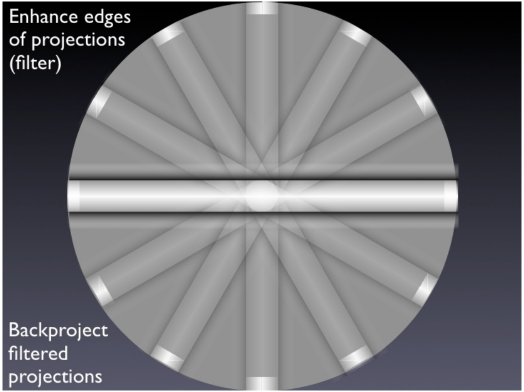
$$\mathrm{ACF} = \exp\left(\mu \frac{D}{2}\right)$$

The corrected intensity I_{corr} is then calculated by evaluating:

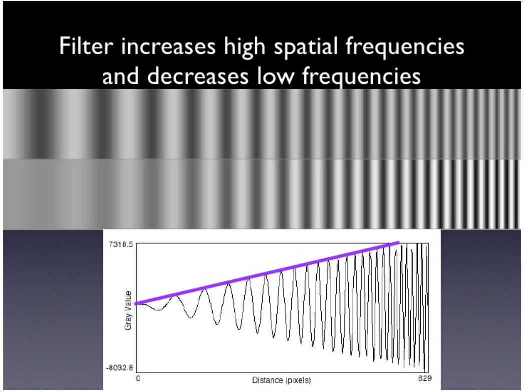
$$I_{\mathrm{corr}} = \mathrm{ACF} \times I_G$$

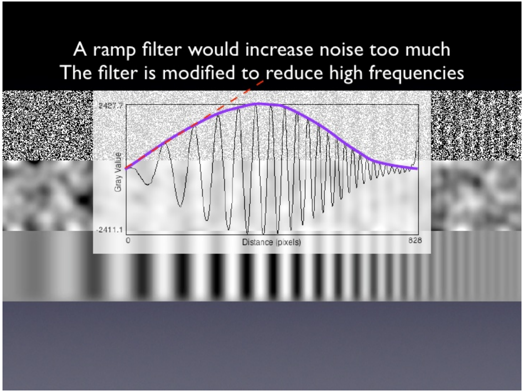
Combination of projections



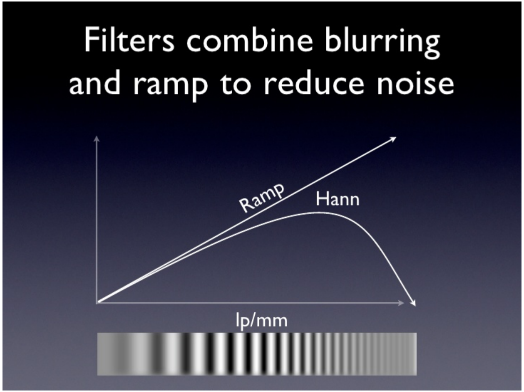


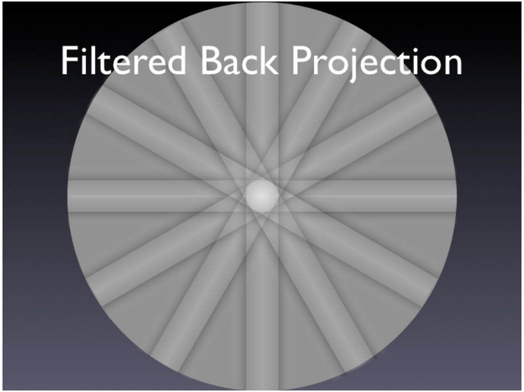
Filtered back projection





Filtered back projection





Summary of section 2

Conjugate counting is exploited to allow attenuation correction

Planar images acquired at a large number of angles are exploited using "back projection" technique to reconstruct 2D image

2D image obtained using back projection is used to derive attenuation correction

"Filtered back projection technique" is used to enhance contrast and reduce noise of 2D image