

# Nuclear diagnostics and Magnetic Resonance Imaging

## Week 2; Lecture 4; Single photon emission computed tomography

**K. Long** ([k.long@imperial.ac.uk](mailto:k.long@imperial.ac.uk))

Department of Physics, Imperial College London/STFC

**R. McLauchlan** ([ruth.mclauchlan@nhs.net](mailto:ruth.mclauchlan@nhs.net))

Radiation Physics & Radiobiology Department, Imperial College Healthcare NHS Trust

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## Section 1

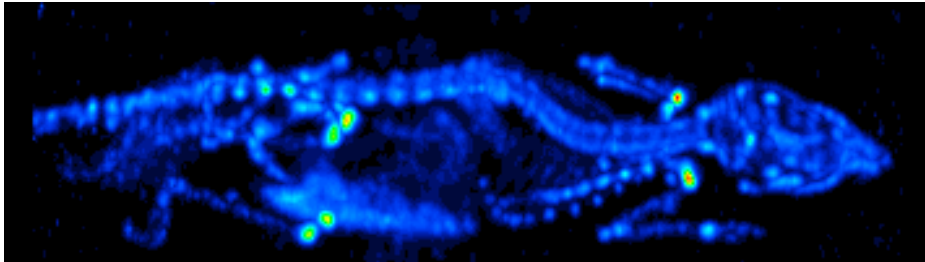
# Introduction

# Introduction

Gamma camera gives single projected image of object; cf conventional x-ray image

**SPECT**: Single Photon Emission Computed Tomography; cf X-ray CT

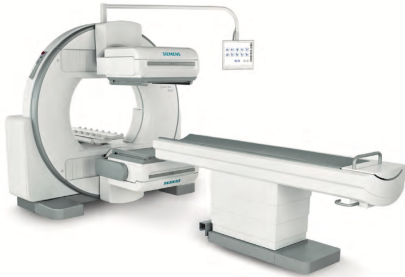
*SPECT image of mouse with bone tracer*



[Click here for C. Lackas' animated gif on wikipedia](#)

Image prepared by C. Lackas

# Typical SPECT systems



Two (or more) gantry-mounted gamma cameras:

- Gamma cameras rotate around patient; 2D cross section
- Images taken from multiple angles
- Bed moves in longitudinal direction

Allows 3D images to be reconstructed

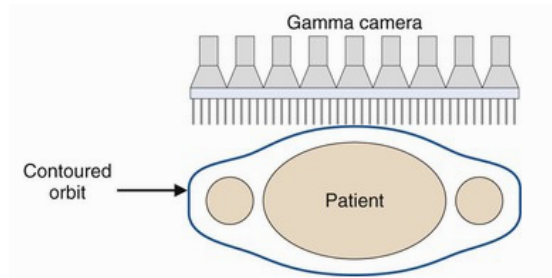
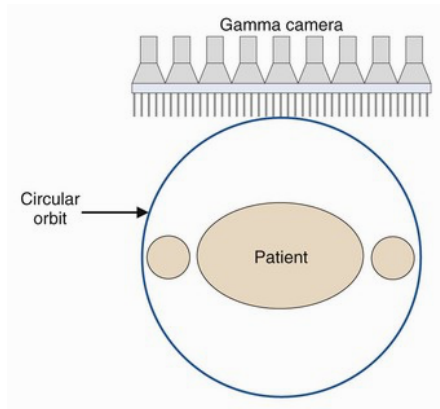


Ring of planar or pinhole gamma cameras

- 2D images obtained without rotation of detectors
- Images taken from multiple angles at the same time
- Bed moves in longitudinal direction

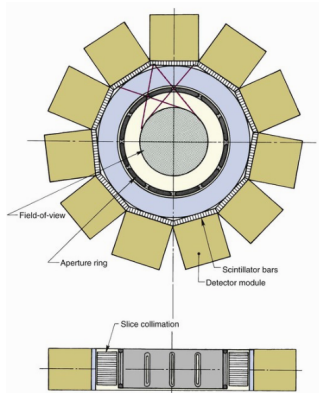
Allows 3D images to be reconstructed

# Circular and contoured orbits

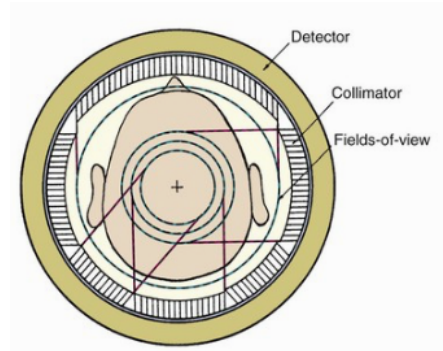


“Elliptical” orbit is more complicated but has the advantage of increased precision

# Alternative configurations, in this case for brain scans



Aperture ring (12 slits) rotates



Each collimator section has its own field-of-view diameter

# Typical parameters

- 64 to 128 angular views
- 2-3 mm linear sampling along longitudinal axis
- 360° data collection
- Reconstructed on  $64 \times 64$  or  $128 \times 128$  matrix
- Field of view  $\sim 40\text{--}60$  cm transaxially
- Stack of images covering  $\sim 30\text{--}40$  cm longitudinally



# Summary of section 1

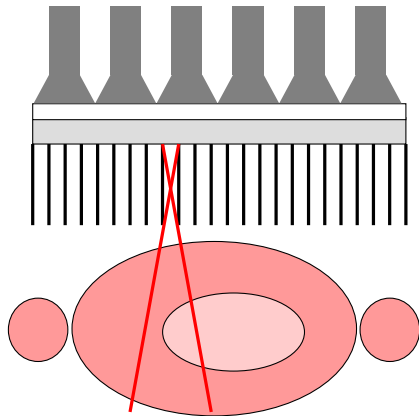
SPECT exploits gamma-camera technology to generate 3D CT image

System either exploit two, conjugate gantry-mounted gamma cameras or a ring of gamma cameras

## Section 2

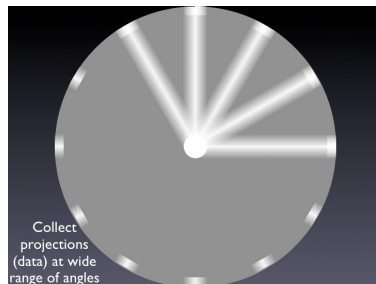
# Reconstruction

## Projection on image plane

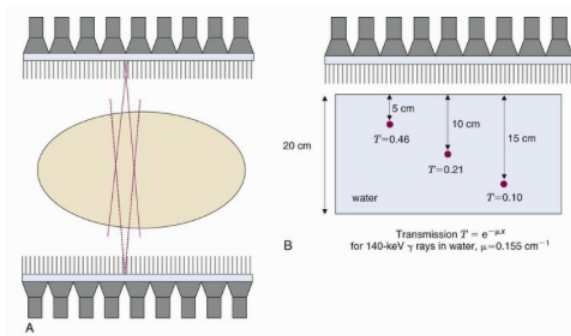


Absorptive collimator means that each hole views a pencil-like area of the object

Repeat for a wide range of angles:



# Geometric response; conjugate counting



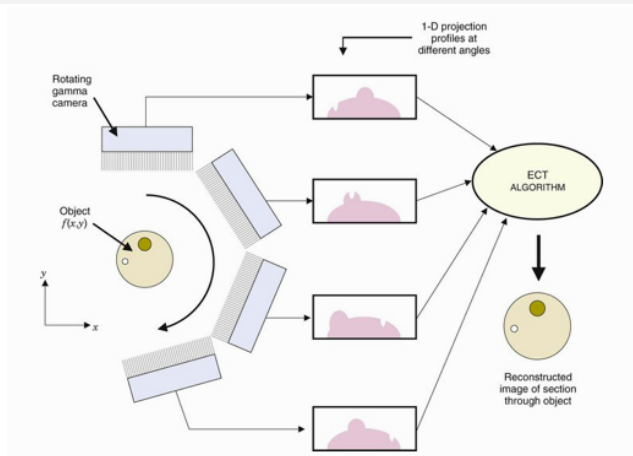
$$I(r, \phi) \neq I(r, \phi + \pi)$$

- Attenuation of  $\gamma$  intensity
- Divergence of image cone
- Measurements at  $\phi$  and  $\phi + \pi$  “conjugate”

Example of attenuation of  $\gamma$ s from  $^{99m}\text{Tc}$ :

- Exploit conjugate measurements to correct for lost attenuation

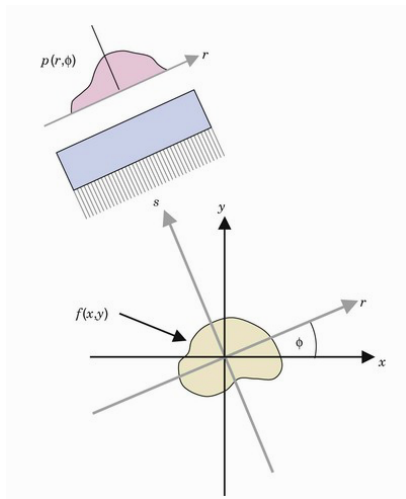
# Back projection



ECT: Emission computed tomography

TCT: Transmission computed tomography

# Back projection: local coordinate system



Local coordinate system  $r, s$ :

- $r$ : coordinate along gamma camera
- $s$ : distance camera to source

$r, s$  coordinates related to  $x, y$  by:

$$r = x \cos \phi + y \sin \phi$$

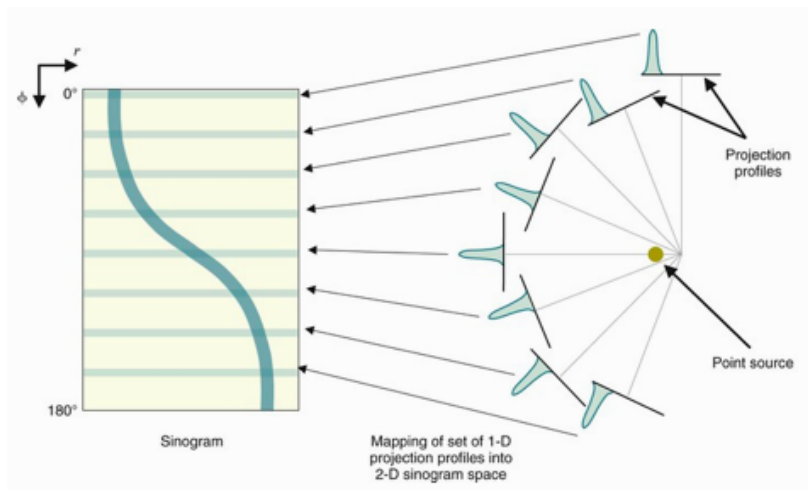
$$s = -x \sin \phi + y \cos \phi$$

$x, y$  may be reconstructed using:

$$x = r \cos \phi - s \sin \phi$$

$$y = r \sin \phi + s \cos \phi$$

# Back projection: sinogram



## Back projection: profile construction

Measurement at each projection measures a response “profile”,  $p(r, \phi)$

Want to reconstruct the activity in a particular slice,  $f(r, s)$  or  $f(x, y)$

In “simple” back projection, the total response measured at a particular  $r_i, \phi_i$  is divided between the pixels along the projected coordinate  $s$

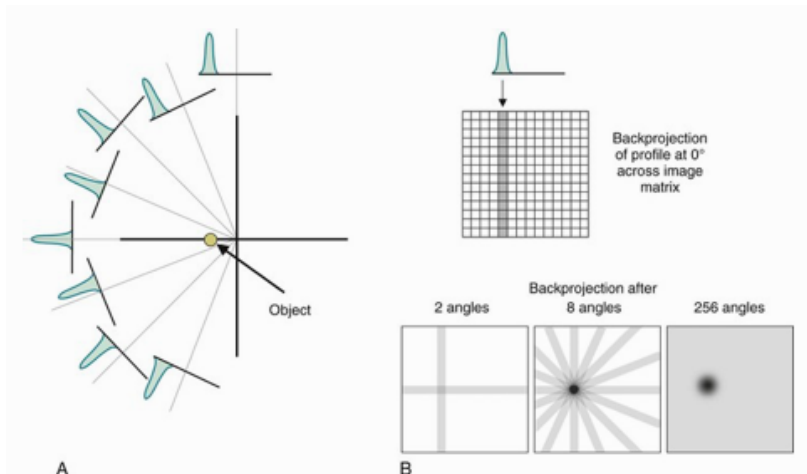
The total (uncorrected) activity,  $f'(x, y)$  within a pixel at coordinate  $x, y$  is then given by:

$$f'(x, y) = \frac{1}{N} \sum_{i=1}^N p(r_i, \phi_i)$$

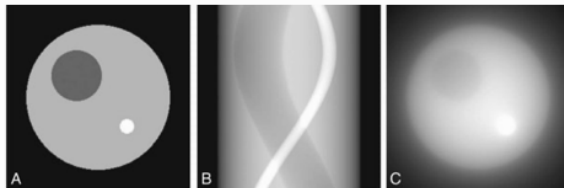
where the sum runs over the  $N$  projections that illuminate the pixel at  $x, y$



# Back projection: illustration



## Back projection: illustration using a simple phantom



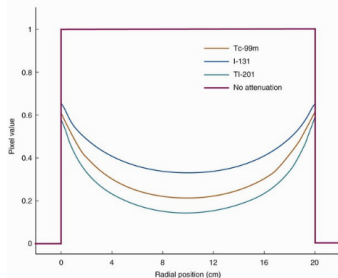
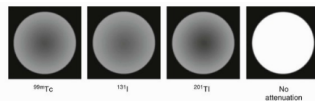
Main features of phantom appear in the image ... but ...

The attribution of activity to pixels along the projected coordinate  $s$  causes “spoke-like” image with few projections

More projections improve image, but, attribution of activity leads to apparent activity outside the object and blurring of the image

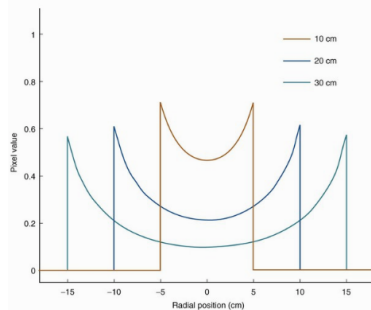
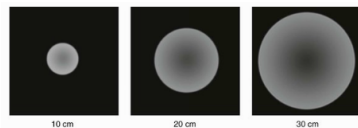
More sophisticated reconstruction algorithms (e.g. filtered back projection, see later) have been developed to overcome this defect

# Attenuation depends on $\gamma$ energy and depth

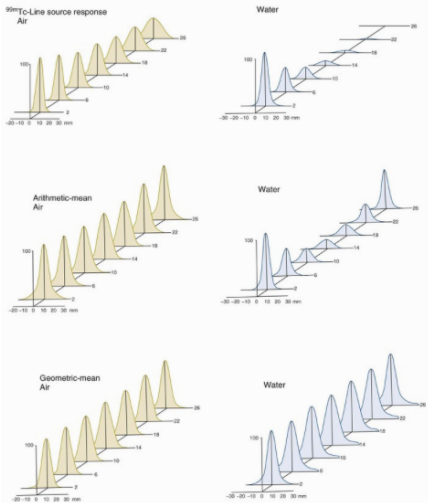


$$E_{\gamma}^{99m\text{Tc}} = 140 \text{ keV}; E_{\gamma}^{131\text{I}} = 364 \text{ keV};$$

$$E_{\gamma}^{201\text{Tl}} = 70 \text{ keV}$$



# Attenuation

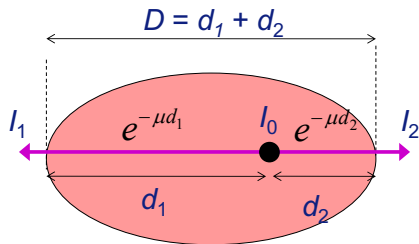


Example:

- High-resolution pin-hole collimator
- Resolution for line source diameter 2.5 mm:
  - As a function of distance source→detector
  - In air (left) and in water (right)
- Corrections applied:
  - Top: no correction
  - Middle: arithmetic mean:
 
$$I_A = \frac{1}{2}(I_1 + I_2)$$
  - Bottom: geometric mean:
 
$$I_G = (I_1 \times I_2)^{\frac{1}{2}}$$

Geometric mean gives most uniform response

# Geometric mean



Intensity measured in two conjugate PMTs,  
numbered 1 and 2:

$$I_1 = I_0 \exp(-\mu d_1)$$

$$I_2 = I_0 \exp(-\mu d_2)$$

Geometric mean;  $I_G$ :

$$\begin{aligned} I_G^2 &= I_0 \times I_0 \\ &= I_0 \times I_0 \exp(-\mu(d_1 + d_2)) \end{aligned}$$

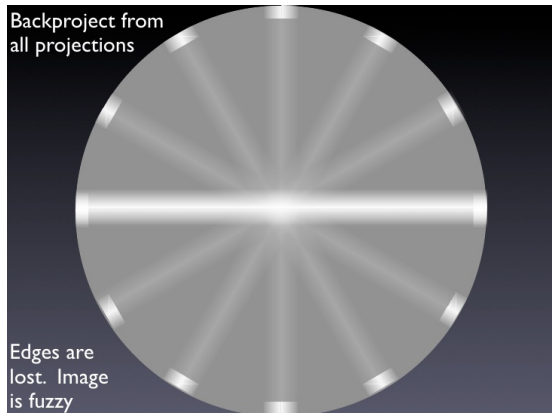
If  $I_0 = I_0$ :

$$I_G = I_0 \exp\left(-\mu \frac{D}{2}\right)$$

i.e.  $I_G$  depends on total depth  $D$  rather than  $d_1$  or  $d_2$ .

The result is exact only for homogeneous media and point sources. Corrections can be derived to accommodate these effects.

# Attenuation correction



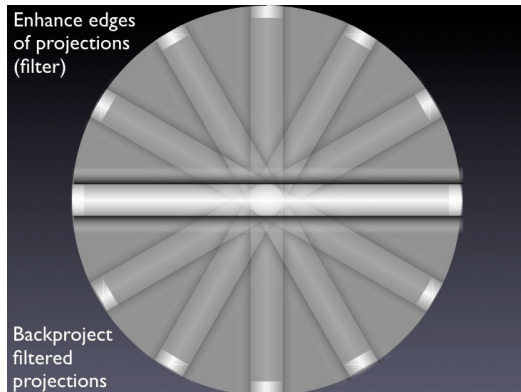
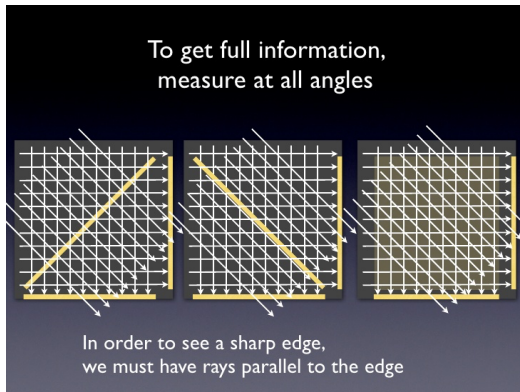
Define, attenuation correction factor, ACF:

$$\text{ACF} = \exp\left(\mu \frac{D}{2}\right)$$

The corrected intensity  $I_{\text{corr}}$  is then calculated by evaluating:

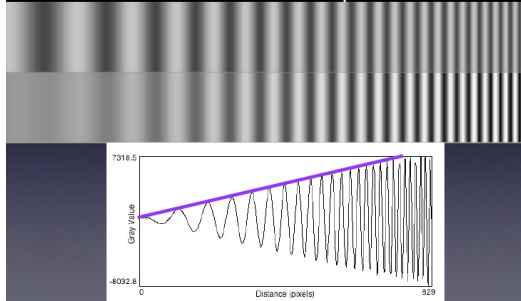
$$I_{\text{corr}} = \text{ACF} \times I_G$$

# Combination of projections

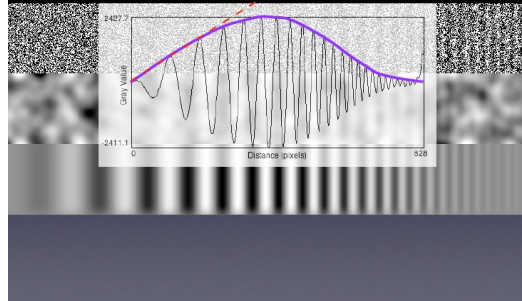


# Filtered back projection

Filter increases high spatial frequencies and decreases low frequencies



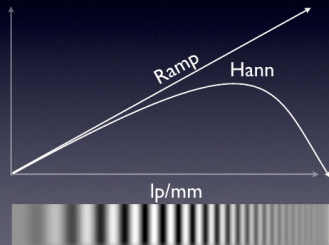
A ramp filter would increase noise too much  
The filter is modified to reduce high frequencies



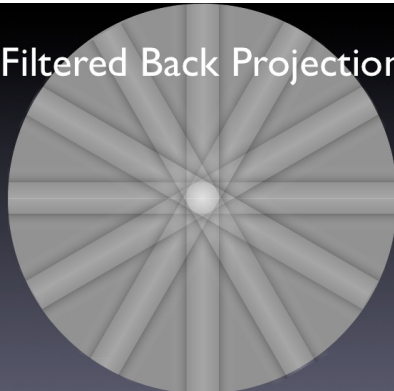


# Filtered back projection

Filters combine blurring and ramp to reduce noise



Filtered Back Projection



## Summary of section 2

Conjugate counting is exploited to allow attenuation correction

Planar images acquired at a large number of angles are exploited using “back projection” technique to reconstruct 2D image

2D image obtained using back projection is used to derive attenuation correction

“Filtered back projection technique” is used to enhance contrast and reduce noise of 2D image

## Section 3

# Attenuation correction

# Attenuation correction strategies

- ① Exploit ACF in “Chang’s multiplicative method”
- ② Generate a transmission map using “attenuation scans”
- ③ Use mean patient shape
  - Disadvantage “there is no mean (or average) patient”
- ④ Exploit CT image:
  - X-ray image processed to give transmission map that can be used to calculate ACF as a function of position

Will consider 1 and 2 below

# Chang's multiplicative method

Steps:

- 1 Reconstruct image without any attenuation correction
- 2 Use reconstructed image to identify contour of patient
- 3 Assume uniform linear attenuation coefficient,  $\mu$ , and calculate ACF pixel by pixel
- 4 ...

## Calculation of ACF pixel by pixel

For pixel at position  $x, y$ , a distance  $d_i$  from the surface in the direction of the camera, the pixel's attenuation factor,  $\eta_i$ , is given by:

$$\eta_i = \exp(-\mu d_i)$$

For a pixel at  $x, y$ , can now sum attenuation over all pixels between  $x, y$  and the surface to obtain the total attenuation factor for the path:

$$\eta = \frac{1}{N} \sum_1^N \exp(-\mu d_i)$$

As before,  $N$  is the number of projections. The attenuation correction coefficient, now a function of  $x$  and  $y$  is given by:

$$\text{ACF}(x, y) = \frac{1}{\frac{1}{N} \sum_1^N \exp(-\mu d_i)}$$

# Chang's multiplicative method

Steps:

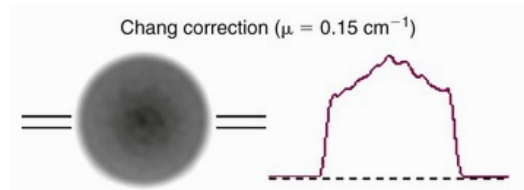
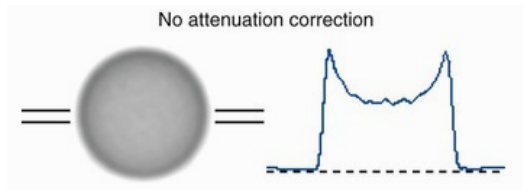
- 1 Reconstruct image without any attenuation correction
- 2 Use reconstructed image to identify contour of patient
- 3 Assume uniform linear attenuation coefficient,  $\mu$ , calculate  $ACF(x, y)$
- 4 Apply ACF pixel by pixel:

$$f(x, y) = f'(x, y) \times ACF(x, y)$$

where  $f'(x, y)$  is the uncorrected response reconstructed in the pixel at  $x, y$ , and  $f(x, y)$  is the corrected response.

## Example

20 cm diameter cylinder with uniform concentration of  $^{99m}\text{Tc}$ .

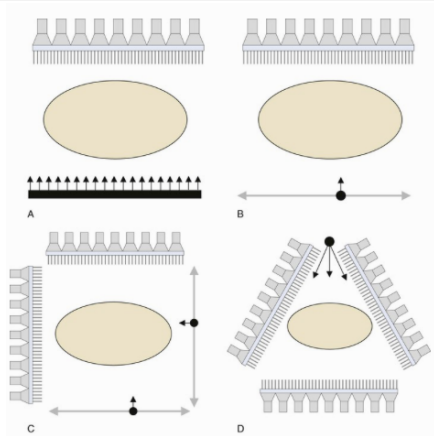


Apparent “over correction” attributed to scattered events.

In this example Chang's method has been applied, followed by a further correction by “forward projecting”. The corrected image is projected to the gamma camera. The predicted response of the camera is then compared to the measured response and a further correction is made based on the difference of the forward projection and the measurement.



# Transmission scans



A: Flood source  
B: Single source

C: 2 orthogonal sources  
D: Stationary line source

Reference scan:  $I_{\text{ref}}$ ; transmission scan:  $I_{\text{trans}}$   
For a particular projection element:

$$I_{\text{trans}} = I_{\text{ref}} \exp(-\mu d)$$

Taking the logarithm of the ratio:

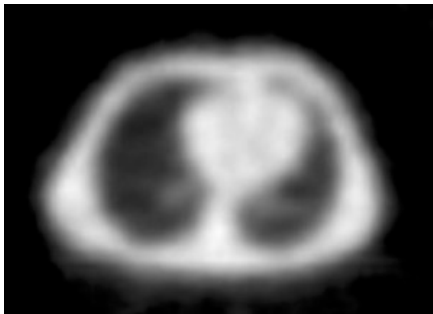
$$\ln \left( \frac{I_{\text{ref}}}{I_{\text{trans}}} \right) = \mu d$$

Back-projection technique yields

$$\mu d = \sum_i \mu_i d_i$$

where the  $i^{\text{th}}$  pixel is of size  $d_i$  and is characterised by  $\mu_i$

## Transmission scan: example



Transmission map of thorax using moving line source.

Radionuclides for transmission scans:

- $^{99m}\text{Tc}$  ( $E_\gamma = 140 \text{ keV}$ )
- $^{153}\text{Gd}$  ( $E_\gamma = 97 \text{ keV}$  and  $103 \text{ keV}$ )
- $^{123}\text{Te}$  ( $E_\gamma = 159 \text{ keV}$ )

Long half-life convenient as then source does not need to be replaced frequently

## Summary of section 3

Correction for attenuation an important step in SPECT image-reconstruction process

A variety of procedures are used in practice; Chang's multiplicative method and the transmission-scan method were summarised

## Section 4

# Scattering correction

# Scatter correction

Primarily due to Compton scattering

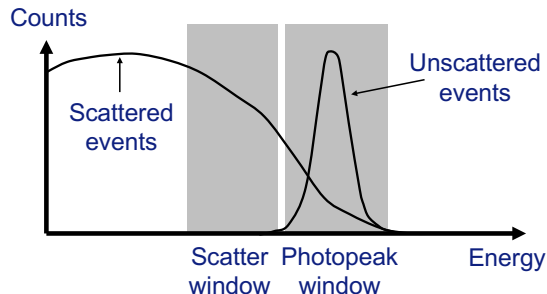
Effect is smaller in magnitude than attenuation

Ratio of scattered to non-scattered photons may be as high as 40%, even when using a narrow energy window

Scatter reduces image contrast as events are put in the “wrong place” and leads to an overestimation of radioactivity in a pixel

Loss of contrast may obscure clinically relevant details

# Scatter correction



Weighting factor must be determined experimentally, it depends on:

- Choice of energy detection window (photopeak window)
- Size of object being scanned
- Energy resolution of gamma camera

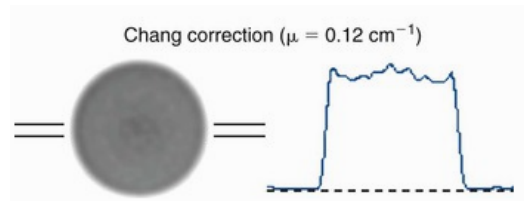
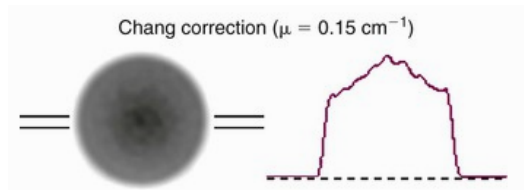
Estimate contribution of scattering events in “photopeak window” by calculating a weighting factor,  $w_f$

Number of events subtracted from photopeak is  $w_f$  times number of events in scatter window

Scatter-correction method limited by differences in spatial distribution of scatter and photopeak

## Example of impact of scatter correction

20 cm diameter cylinder with uniform concentration of  $^{99m}\text{Tc}$ .



“Over correction” noted above removed by scatter correction

## Summary of section 4

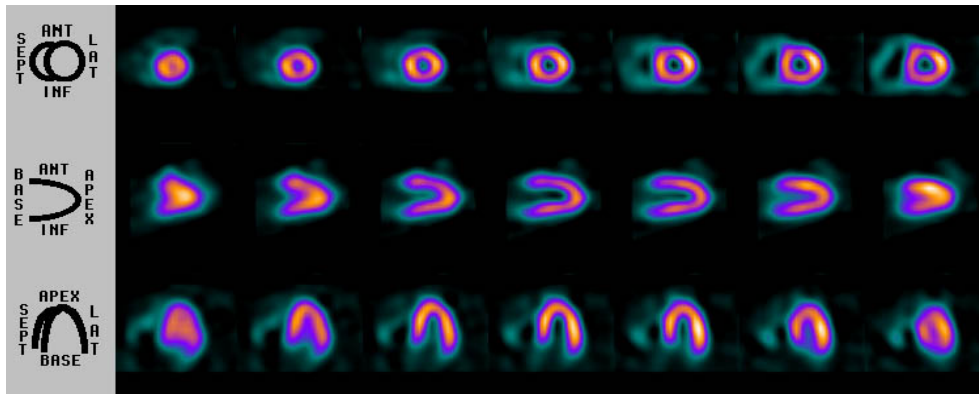
Weighting factor calculated by “extrapolating” contribution from Compton-scattered photons in region of “photo-peak window”



## Section 5

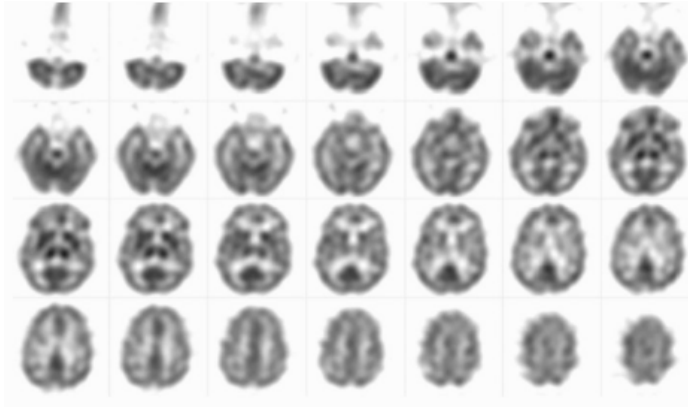
# Examples

# SPECT images of cardiac perfusion



Cardiac perfusion scan using  $^{99m}\text{Tc}$ -sestamibi. Images are shown in three slices, as indicated on the LHS. The time sequence (left to right) is in steps of 20 s.

# SPECT images of brain perfusion



Brain perfusion scan using  $^{99m}\text{Tc}$ -HMPAO. Images were acquired with an exposure of 40 s per view.