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# Nuclear diagnostics and Magnetic Resonance Imaging

## Week 2; Lecture 5; Section 1: Principles of positron emission tomography

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Section 1

# Principles of positron emission tomography

# Positron Emission Tomography; the process

PET exploits photons generated in annihilation: $e^+ + e^- \to \gamma_1 + \gamma_2$

$\beta^+$ from decay scatters elastically off atomic electrons, loosing energy, until it annihilates

Annihilation assumed to be at rest. To conserve energy and momentum:

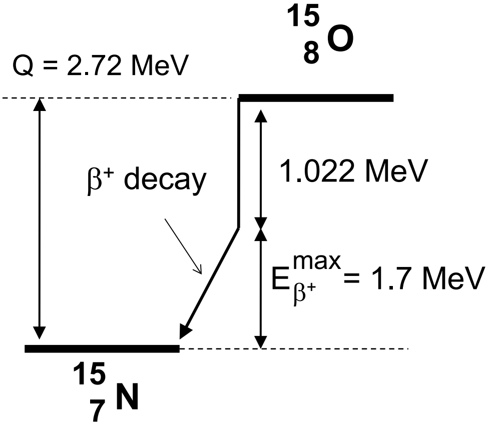
- Photons produced back-to-back
- Photon energies equal: $E_{\gamma_1} = E_{\gamma_2} = E_\gamma = mc^2 = 511\,\mathrm{keV}$

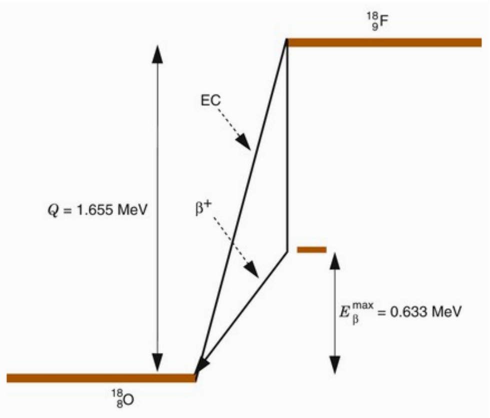
Back-to-back topology localises annihilation signal to a line in 3D space

PET detectors use inorganic scintillators with large $Z$:

- $E_\gamma$ large compared to photons used in SPECT
- So require dense scintillator with greater "stopping power" than NaI

# Beta($^+$) decay, reprise by example



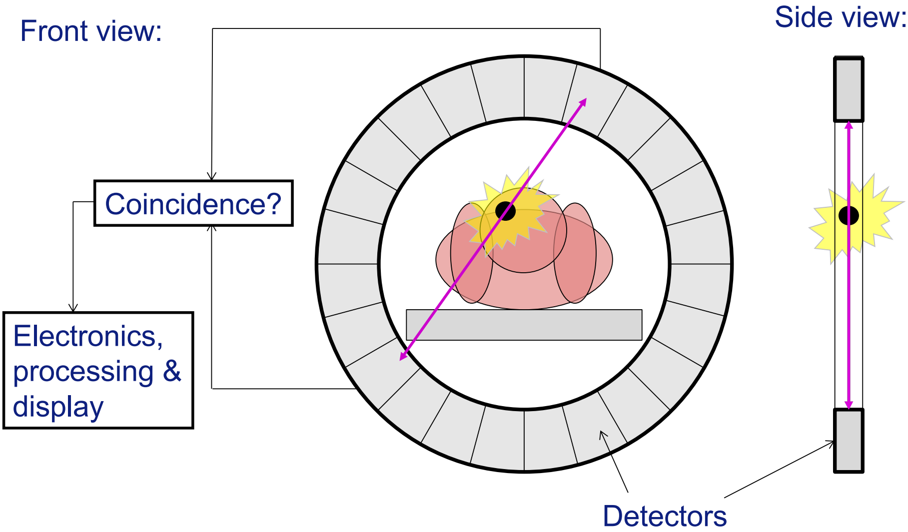

$${}^{15}_{8}\mathrm{O} \rightarrow {}^{15}_{7}\mathrm{N} + e^{-} + e^{+} + \nu_e$$

$${}^{18}_{9}\mathrm{F} \rightarrow {}^{18}_{8}\mathrm{O} + e^{-} + e^{+} + \nu_e$$

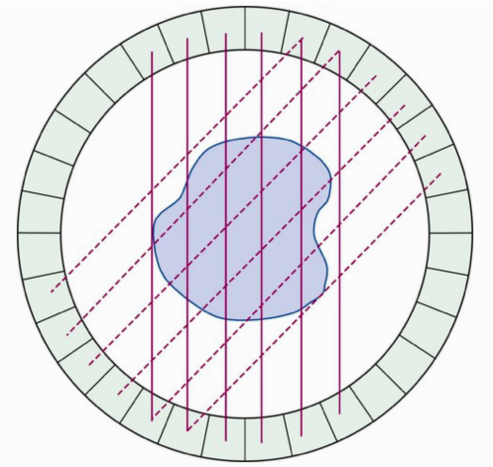
# Positron emitting radionuclides

| Isotope | Half-life | $\beta^+$ fraction | Max. kinetic energy | Average positron range in water (mm) |
|---|---|---|---|---|
| C–11 | 20.3 mins | 0.99 | 0.96 MeV | 1.0 mm |
| N–13 | 9.96 mins | 1.00 | 1.19 MeV | 1.3 mm |
| O–15 | 123 secs | 1.00 | 1.72 MeV | 2.0 mm |
| F–18 | 110 mins | 0.97 | 0.64 MeV | 0.6 mm |
| Ga–68 | 68.3 mins | 0.88 | 1.90 MeV | 1.2 mm |
| Rb–82 | 78 secs | 0.95 | 3.35 MeV | 2.8 mm |

# Principle


Front view:
Side view:
Coincidence?
Electronics, processing & display
Detectors

# Taking views in parallel



Multiple projections taken at the same time:

- Schematic shows two projections
- Ring of detectors can take all projections simultaneously

  $\rightarrow$ An advantage over SPECT

"Annihilation Coincidence Detection" (ACD)

- ACD localises events to a line; "electronic collimation"

Eliminates need for absorptive septa

- Enhances geometrical efficiency substantially

  $\rightarrow$ Another advantage over SPECT

# "Block detector" for PET

Cuts in scintillator:

- Do not extend to full depth
- Reflective material fills gaps

Light yield function of position

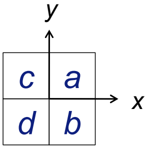
Example of "multiplexing";
Reduces cost of optical readout

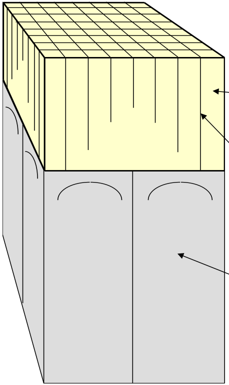
a, b, c, d are the number of counts in the 4 PMTs

$\ell$ is the size of the PMT (approximated as the side of a square)

$$X = \ell(a + b - c - d) / (a + b + c + d)$$

$$Y = \ell(a - b + c - d) / (a + b + c + d)$$



Segmented block of BGO or LSO scintillator

Reflector material between segments

Four single channel photomultiplier tubes

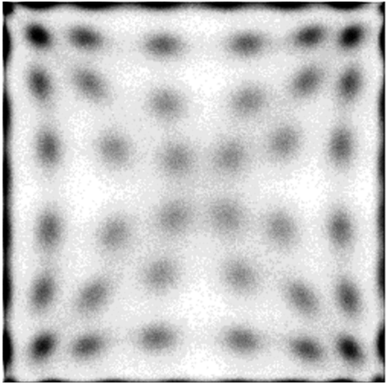
N.B. in the formula on this slide, *X* and *Y* are measured relative to the *centre* of block of 4 PMTs

Example:
64 crystals, 4 pmts:
**multiplexing factor of 16**

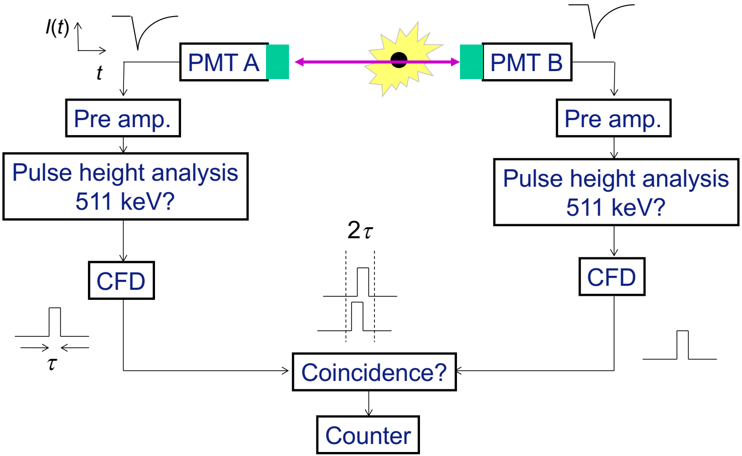
# “Block detector” for PET

Flood irradiation of block detector with 511 keV $\gamma$s:

- Spatial localisation of energy deposits
- Non-linear response corrected with “look-up table”

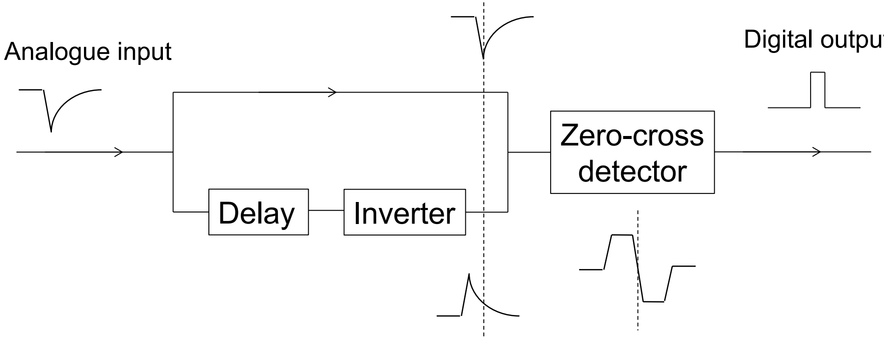


# Forming the coincidence




CFD: constant fraction discriminator

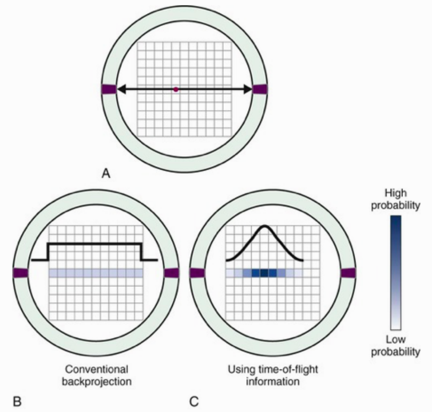
# Constant fraction discriminator (CFD)




Objective: determine arrival time of pulse that is largely independent of pulse height

- Time at which signal reaches a fraction (e.g. 25%) of its peak amplitude

# Time-of-flight measurement




A: ACD configuration
B: No ToF: back-projection with equal probability
C: ToF: back-projection localised at $\Delta d$

If time-of-arrival difference is $\Delta t$, then:

$$\Delta d = \frac{c\Delta t}{2}$$

where $\Delta d$ is measured w.r.t. the midpoint

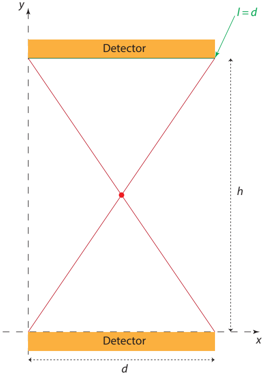
Localises back projection to $\Delta d \pm \sigma_{\Delta d}$

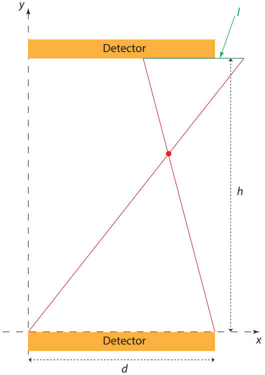
Requires fast scintillator, fast electronics to yield $\Delta t \sim 100 - 200$ ps

Leads to $\sigma_{\Delta d} \sim 2 - 3$ cm

Of benefit in removal of ambiguities in reconstruction

# Intrinsic spatial resolution

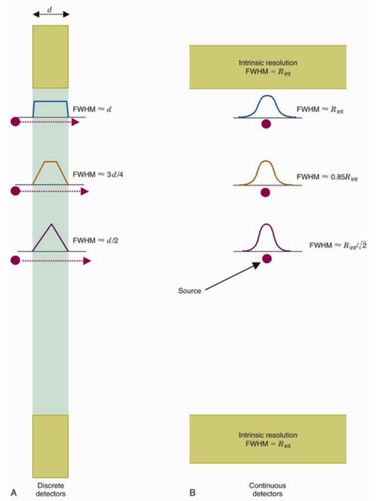



Source at $\left(x = \frac{d}{2}, y = \frac{d}{2}\right)$;
all events in cone accepted

Source at $(x, y)$;
accept only events striking both detectors

# Intrinsic spatial resolution




With coordinates defined above, projected length $l$ is given by:

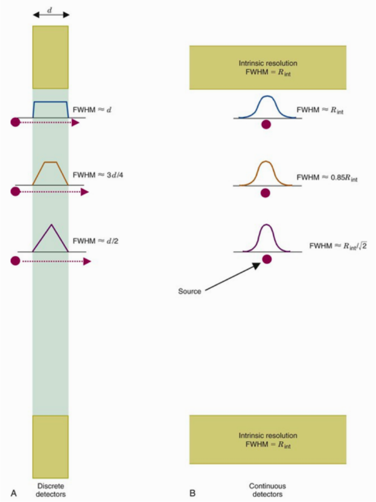
$$l = d\frac{h-y}{y}$$

Observations:

- At fixed $y$, $l$ is independent of $x$
- Intensity recorded is a function of $x$
- At fixed $y$:
  - PDF rises from 0 at $x = 0$ and $x = d$
  - Plateau in PDF reached when $x_{\rm P} = \frac{d}{h}y$
- For $y = \frac{h}{2}$ no plateau because $x_{\rm P} = \frac{d}{2}$

Shown in LH column of figure

# Intrinsic spatial resolution



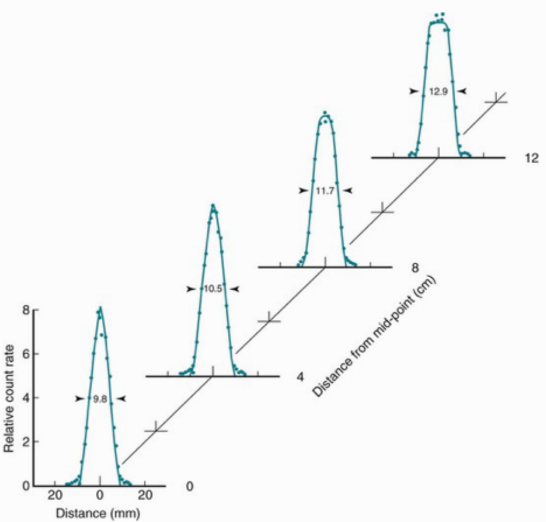

Referring now to RH column of figure . . .

Define FWHM at $y = h$: $\mathrm{FWHM} = R_{\mathrm{int}}$

Intrinsic resolution:

- $y = h$: rectangular distribution
  $$\sigma_{\mathrm{int}} = \frac{R_{\mathrm{int}}}{2} = \frac{d}{\sqrt{12}}$$
- $y = \frac{h}{2}$: triangular distribution
  $$\sigma_{\mathrm{int}} = \frac{R_{\mathrm{int}}}{2\sqrt{2}} = \frac{d}{\sqrt{2}\sqrt{12}}$$
- Intermediate $y$: "truncated triangle"
  $$\frac{d}{\sqrt{2}\sqrt{12}} \le \sigma_{\mathrm{int}} \le \frac{d}{\sqrt{12}}$$

# Measured intrinsic resolution




Measured "residuals" for:

- 2 detectors each with $d = 17$ mm
- $\sigma_{\text{int}} = 4.91$ mm

  $\Rightarrow \mathrm{FWHM} = R_{\text{int}} = 9.81$ mm

Resolution favourable cf SPECT with:

- Conjugate sampling
- Arithmetic mean position estimation

For equivalent position resolution PET is:

- More efficient that SPECT (collimator)
- Faster; all images taken at once

# Summary of section 1

PET exploits kinematics of $e^{+}e-$ annihilation using coincident, conjugate detection to localise position of a particular disintegration to a line

Appropriate readout electronics allows multiple projections to be acquired at the same time; an improvement in efficiency compared to SPECT

Time-of-flight measurement used to localise position of disintegration along line defined annihilation-photon pair

Spatial resolution varies slowly across fiducial volume