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Magnetic Resonance Imaging

Week 5; Lecture 10; Section 2: Encoding spatial information: 1

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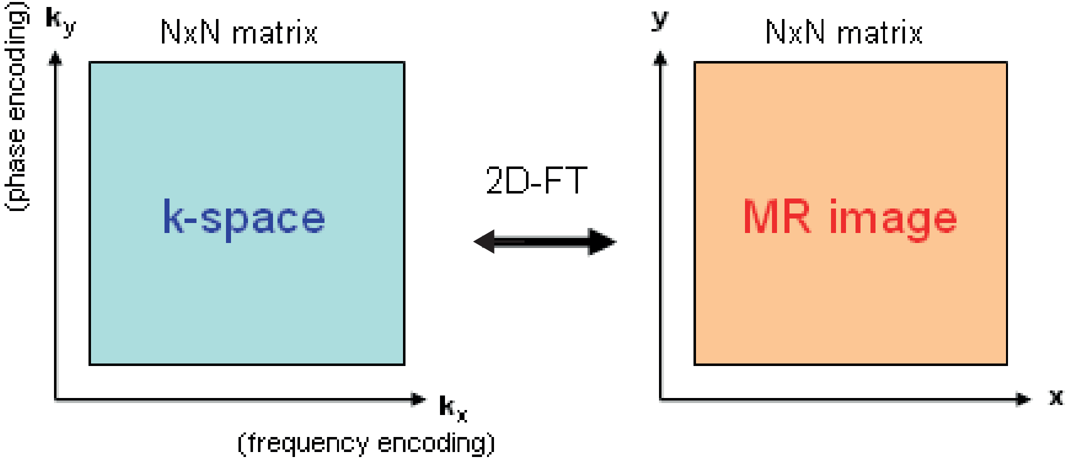
Section 2

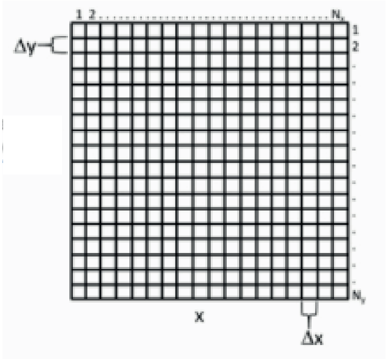
Encoding spatial information in k-space

Encoding spatial information into the net magnetisation

The basis is a 2D Fourier transform:



2D Fourier transformation



2D image in "coordinate space", x, y, presented in pixel grid

Field of view, FOV, in coordinate space:

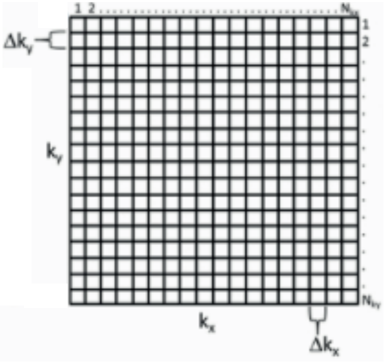
$$(x_{\max} - x_{\min}, y_{\max} - y_{\min})$$

Pixel size (resolution):

$$\Delta x = \frac{x_{\max} - x_{\min}}{N_x}$$

$$\Delta y = \frac{y_{\max} - y_{\min}}{N_y}$$

2D Fourier transformation



2D image in "k space", k_x, k_y, presented in pixel grid

Field of view, FOV, in k space:

$$\left(k_{x\,\mathrm{max}} - k_{x\,\mathrm{min}}, k_{y\,\mathrm{max}} - k_{y\,\mathrm{min}}\right)$$

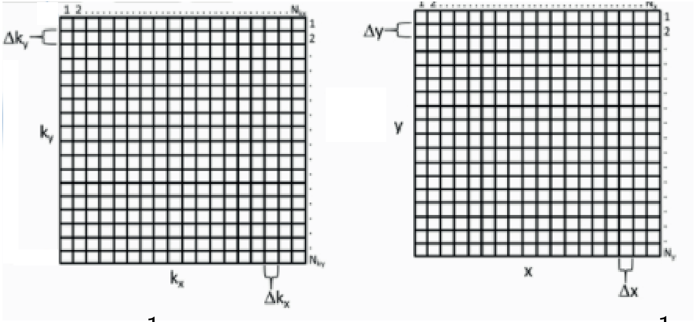
Pixel size (resolution):

$$\Delta k_x = \frac{k_{x\,\mathrm{max}} - k_{x\,\mathrm{min}}}{N_x}$$

$$\Delta k_y = \frac{k_{y\,\mathrm{max}} - k_{y\,\mathrm{min}}}{N_y}$$

2D Fourier transformation

Transformation between resolution in coordinate-space and *k*-space representations:



$$\Delta k_x = \frac{1}{(x_{\max} - x_{\min})} \qquad \Delta x = \frac{1}{(k_{x\,\max} - k_{x\,\min})}$$

$$\Delta k_y = \frac{1}{(y_{\max} - y_{\min})} \qquad \Delta y = \frac{1}{(k_{y\,\max} - k_{y\,\min})}$$

2D Fourier transformation

Define $\rho(x, y)$ to be the intensity pixel-by-pixel in coordinate space.

2D Fourier transform from coordinate to k space is then:

$$S\left(k_x, k_y\right) = \int_{y_{\min}}^{y_{\max}} \int_{x_{\min}}^{x_{\max}} \rho\left(x, y\right) \exp\left(-i2\pi k_x x\right) \exp\left(-i2\pi k_y y\right) dxdy$$

where $S(k_x, k_y)$ is the intensity pixel-by-pixel in k space

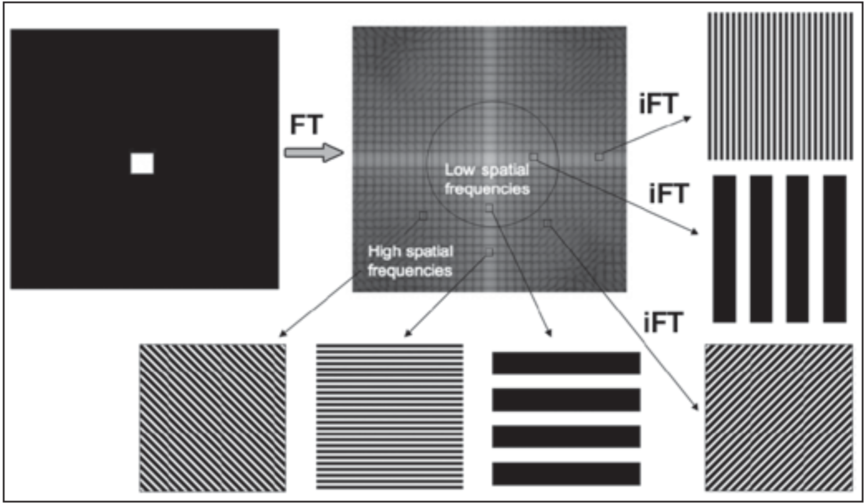
Inverse Fourier transform takes k-space intensity map to coordinate-space intensity map:

$$\rho\left(x, y\right) = \int_{k_{y\,\min}}^{k_{y\,\max}} \int_{k_{x\,\min}}^{k_{x\,\max}} S\left(k_x, k_y\right) \exp\left(i2\pi k_x x\right) \exp\left(i2\pi k_y y\right) dk_x dk_y$$

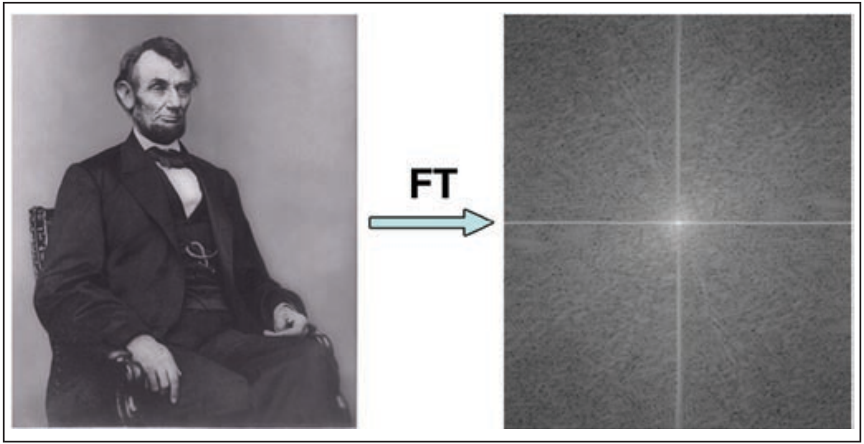
Example one: a single dot

Example two: three dots

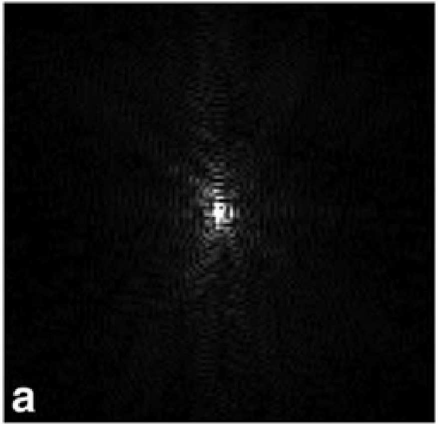
Example three: Square in centre of field of view



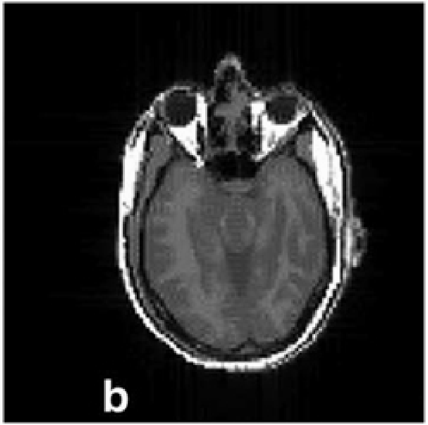
Example three: Abraham Lincoln



Example three: Abraham Lincoln



(a) k-space image of head



(b) coordinate-space image of head

Challenge: record k-space image using NMR signals

Summary of section 2

Intensity distribution in "coordinate space" ($\rho(x, y)$) mapped using a Fourier transform onto intensity distribution in "k"-space ($S(k_x, k_y)$)

Signals generated in MRI scan recorded in k-space; coordinate space image obtained by inverse Fourier transform